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on the productivity cost of illness

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## The Impact of Firms' Expectations & Adjustments on the Productivity Cost of Illness

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**Abstract** Sickness-related absenteeism hinders firms' productivity and reduces output, an effect referred to as *indirect cost* (IC) and often included when assessing the burden of an illness or cost-effectiveness of a treatment. The companies may, however, foresee this risk and modify hiring or contracting policies. We present a model of a firm allowing to estimate IC while accounting for such adjustments. We show that the risk of illness does not change the general shape and properties of the (expected) marginal productivity function. We apply our model to several illustrative examples and show that firm's adjustments impact IC in an ambiguous way, depending on detailed company/market characteristics. Sometimes the company reduces the employment (further increasing IC), yet sometimes the opposite (even generating indirect gains). Contrary to previous literature findings, teamwork and shortfall penalties may reduce IC in some settings. Our analysis highlights that IC should be split into the result of companies *preparing for* and *actually experiencing* sick leaves, at least when friction cost approach is taken. To what extent the former counts as IC may depend on the (labour and good) market structure and the interpretation of equilibrium values. These considerations are usually not addressed in applied IC assessment, which may bias the results.

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## 1 Introduction

Medical technologies are often financed using limited, public budget. Hence, it is important to understand the economic consequences of a disease, e.g., when performing a cost-effectiveness analysis of a health technology in order to decide how to allocate scarce resources. The scope of these consequences depends on whose perspective is taken (Suhrcke et al, 2012; Brouwer et al, 2006). In several countries (e.g., Austria, cf. Walter and Zehetmayr 2006, Sweden, cf. Pharmaceutical Benefits Board 2003, the Netherlands, cf. College voor Zorgverzekeringen 2006, Australia, cf. Pharmaceutical Benefits Advisory Committee 2008, Italy cf. Capri et al 2001) the broadest, societal perspective is recommended. Then, we also need to measure the impact of illness on productivity: sick persons not showing up for work (absenteeism) or showing up but performing poorly due to health problems (presenteeism) constitute an opportunity cost of a disease (how these human resources could be used, were it not for the disease). This is referred to as *indirect cost* (henceforth, IC), see, e.g., Koopmanschap and van Ineveld (1992). IC may also be associated with unemployed persons being ill through the impact on the economic activity of their families (Liljas, 1998). In the present paper we focus on absenteeism only.

Whether and how IC should be estimated is subject to research, Krol et al (2013) present a review of pending questions and prevalent opinions. Briefly, according to the *human capital approach* (HCA) IC should be calculated as the product of the sickness duration (when human capital lies idle) and the value of a day (week, month, etc.) of work (Berger et al, 2001). The latter is often approximated using a daily wage, motivated by the economic proposition that labour is in the equilibrium remunerated at the level of its marginal revenue product (illness is considered to marginally impact the available workforce). In HCA dying before the retirement age results in a long period of human capital not being productive, and so in a substantial IC. The *friction cost method* (FCM) criticizes just that and advocates thinking in terms of actual losses, which are typically mediated, e.g., by companies recruiting new employees to substitute for the unavailable ones (Koopmanschap et al, 1995; Berger et al, 2001). HCA is a recommended approach in Sweden (Pharmaceutical Benefits Board, 2003) and Italy (Capri et al, 2001), while FCM in the Netherlands (College voor Zorgverzekeringen, 2006) and in Australia (Pharmaceutical Benefits Advisory Committee, 2008). In Austria both methods are acceptable (Walter and Zehetmayr, 2006).

Other mechanisms further complicating the picture were brought into discussion in the literature. The diminished productivity may be compensated within a firm by the same individual after recovery or by co-workers; and, contrarily, one person missing may disrupt the work of the whole team (Krol

et al, 2012) or a company may hire temporary agency workers (Bryson, 2013). The strength of these mechanisms may vary between countries due to labour market specificity (e.g., Knies et al, 2013) or even between companies depending on the work organization (e.g. Leigh, 1981, shows that unionisation increases absenteeism, although monopoly wage generates higher opportunity cost of missing work). Pauly et al (2002) present a microeconomic model in which a teamwork (when replacement for the worker cannot be cheaply and immediately found) results in IC being greater than a wage-based estimate. They also discuss the impact of a company having an output target to be met under penalty. Then, again, in their model IC is greater than a wage-based approximation.

While considering the teamwork and output shortfall penalty as Pauly et al (2002), in the present paper we explore a yet different element of the puzzle: companies do not only passively bear the consequences of sick-leaves but rather try to prevent any negative shocks on their productivity. Many companies sponsor, e.g., flu vaccines for employees, so as to reduce winter sick-leaves. Burton et al (2003) reviewed several studies inspecting the impact of various pharmaceuticals (e.g., influenza vaccination and migraine treatment) on productivity losses; the authors suggest that such information should also be used by employers, e.g., when deciding about the purchase of a health care plan. The employers may also consider promoting a health life-style among employees (e.g., Audrey and Procter, 2015; Loepke et al, 2015).

Here we consider companies foreseeing the decreased productivity of employees due sick-leaves and adjusting the total amount of workforce and the amount of goods they contract to deliver. We aim to determine the effect of such adjustments on IC. For that purpose we construct a simple microeconomic model of a company, allowing to account for the sick leaves and the output contracting with shortfall penalties. Our model allows to differentiate between the impact of preparing for sick-leaves (i) and of the actual sick leaves (ii) on the value of overall production; (i) happens via companies adjusting behaviour in the labour and good markets, (ii) results from the reduction of the available workforce. Again, as we are focused on IC, the changes in company's profit are only of secondary importance (obviously, the possibility of sickness can only make companies worse-off, while being able to adjust can only help to alleviate the loss). Thus, for example, in our analysis the output shortfall penalty is interesting only insofar as it impacts the value of production delivered onto the market via companies' hiring and contracting policies.

We apply the model to several illustrative scenarios. Our goal is not to calibrate the model to any specific company, and so the exact values of parameters are of no direct importance and are not interpreted. We rather aim to show the variety of possible situations and discuss prevalent mechanisms. In our model we focus on companies' expectations and adjustments *before* the sick leaves actually occur. For that reason we do not consider, for example, workers replacement as this is rather an *ad hoc* action *after* the illness has occurred.

The present study is a theoretical one (as many others in this line of research, e.g., [Pauly et al 2002](#), [DeLeire and Manning 2004](#), [Arnold and de Pinto 2014](#)), it is then quite natural to ask whether IC has been proved to actually be real. The literature abounds with publications estimating IC for various illnesses, e.g., rheumatoid arthritis ([Fautrel et al, 2011](#)), obesity ([Wolf and Colditz, 1998](#)), depression ([Woo et al, 2011](#)), or diabetes ([Hex et al, 2012](#)). These studies, however, do not prove the losses actually take place, but rather estimate these losses *assuming* they actually happen and can be approximated by some formulae based on economic reasoning. In this context, the research directly showing the existence of IC is interesting, however scarce. [Cockburn et al \(1999\)](#) showed that the difference in actual output (number of claims processed by the employees of the insurance company) differed by ca. 13% depending on the type of antihistamines they were taking. [Orem et al \(2012\)](#) analysed the association between the malaria morbidity and the GDP in Uganda, and it came up negative and statistically significant in econometric modelling. The authors interpret the association as the result of a decreased productivity of workers, an increased labour turnover, and a labour loss in case of worker's death. [Kirigia et al \(2002\)](#) present a similar empirical research, using econometric modelling to measure the impact of HIV/AIDS on the GDP of African countries. The results show that the HIV/AIDS-driven mortality and morbidity reduce GDP on average, the statistical significance was, however, not obtained. The study indicated several factors that may weaken the impact: high unemployment, an ease to replace a worker in a low specialised economy, culture (the non-working family members often taking over the responsibilities of the ill), and the characteristics of the illness (the disease for a substantial part does not cause any symptoms lowering the productivity).

There is also no strong empirical evidence regarding whether and how the companies actually adapt to the possibility of sick leaves and decreased productivity of workers. Still, we can expect that the companies—perhaps not using any modelling and equation-solving but via trial-and-error and facing economic survival-of-the-fittest mechanism—do optimize their behaviour. If not, this paper can always be treated as a description of what would happen in the ideally managed company.

In the next section we present our model and introduce the technical assumptions used in the study. Then, in the third section we apply this model to several, hypothetical scenarios illustrating possible, qualitatively different situations. In [Section 4](#) we discuss our results. [Section 5](#) briefly concludes. We put all the proofs in the appendix.

## 2 The model

Below we present detailed assumptions in our model of a company. We start with a simple, benchmark case with no illnesses and, hence, no uncertainty. Then we present the case where stochastic sick-leaves are possible. Finally, we discuss how we model contracting the output and shortfall penalty.

## 2.1 No sick-leaves

The company maximises its profit. It attains revenues by selling a single type of good, whose price is determined in a perfectly competitive market: company's actions do not influence the price and the company can sell whatever it produces. The product cannot be stored, and so we analyse the functioning of the company in a single period, when this product is manufactured and delivered to the market. The specific length of this period is not specifically defined.

We explicitly only consider labour as a production factor. One interpretation could be that the above-mentioned length of the period is too short for a company to consider changing the amount of capital, and so the cost of capital, being fixed, does not impact the marginal analysis. (We assume, however, that this fixed cost is not so high so as to consume all the profit and make the company want to shut down.)

We measure the quantity of labour in discrete values: 0, 1, 2, ..., interpreted, for example, as a number of full-time workers (or in less coarse a fashion, e.g., a number of half-time workers). We believe that this is much closer to reality than assuming that the company can optimize the amount of labour in a continuous way. We still appreciate, that this may be important for the results as the optimal solution can discontinuously jump between neighbouring values. Additionally notice, that discretizing the amount of labour allows to easily model (in the next subsection) the number of sick leaves using a binomial distribution. The properties of specifically binomial distribution are also used in the proofs of propositions presented below.

We also assume that the labour market is perfectly competitive, and so company's decisions do not impact the wage and the company can hire whatever number of workers it wants. Then, the company's profit,  $\pi(L)$ , is given as:

$$\pi(L) = pY(L) - wL, \quad (1)$$

where:

- $p$  is the price of manufactured product,
- $w$  is unit cost of labour,
- $L$  is the number of employees, and is the only decision variable,
- $Y(L)$  is the level of production achieved by  $L$  employees.

The company maximises its profit by setting the quantity of labour  $L$  so as to maximize equation 1. The additional product resulting from hiring  $L$ -th employee is denoted by  $MP(L) = Y(L) - Y(L - 1)$ , where we take  $Y(0) = 0$ .

In the paper we consider only unimodal functions  $MP(L)$  of two types: either decreasing in  $L$ , denoting the declining marginal productivity of labour, i.e., each consecutive employee contributing less and less to the overall product (cf. Figs. 1&2); or first increasing and then decreasing in  $L$ , denoting that the company first requires some minimal amount of employees so as to operate effectively but then is subject to diminishing productivity of labour (cf. Figure 3). The former may describe companies delivering simple products and

services, while the latter may characterize companies delivering more complicated or complex goods that require cooperation between some minimal number of team members. The latter type thus represents teamwork in our model.

Let  $L_h^*$  denote the optimal level of employment in this no-illness (all-healthy) case. It is easy to see that either  $L_h^*$  will be equal to the greatest  $L$  for which  $MP(L)p \geq w$  (as long as the overall revenue exceeds cost, i.e.,  $\pi(L_h^*) \geq 0$ ) or the company will quit the market setting  $L_h^* = 0$ . Taking a non-strict inequality we assume here, with no impact on the examples we consider, that the company is also willing to hire a person if the value of additional product just covers the wage.

## 2.2 Sick-leaves

We now introduce the possibility of employees being ill and absent. We assume that during the whole considered period every employee is either healthy or sick. Parameter  $s$ ,  $0 \leq s < 1$ , denotes the probability of illness, and we assume  $s$  is known (e.g., based on historical data) and identical for all the employees. We also assume that individual employees being ill are independent events (and so we do not consider contagious diseases as, e.g., [Colombo et al 2006](#) or [Kamiński et al 2010](#) do).

We assume that the company is risk neutral and so maximises the expected profit. The company pays wages also to the ill employees. The expected total product generated with  $L$  employees hired is now given as:

$$Y^E(L) = \underbrace{\sum_{i=0}^L}_{\text{(I)}} \underbrace{\binom{L}{i} s^{L-i} (1-s)^i}_{\text{(II)}} \underbrace{Y(i)}_{\text{(III)}}, \quad (2)$$

where

- (I) index  $i$  goes over all possible numbers of workers reporting for work,
- (II) measures the probability of exactly  $i$  workers being healthy,
- (III) denotes the total product attained with  $i$  workers.

The expected profit of hiring  $L$  workers is then given by:

$$\pi^E(L) = p \sum_{i=0}^L \binom{L}{i} s^{L-i} (1-s)^i Y(i) - wL. \quad (3)$$

We can continue using equation 3 while changing model parameters: work organization and effectiveness (the shape of  $Y(\cdot)$ ), prices and wages ( $p$  and  $w$ ), the probability of being ill ( $s$ ). For given values of parameters, we denote by  $L_s^*$  the optimal number of workers. It is easier, however, to analyse the impact of illnesses using marginal product. Let  $MP^E(L)$  denote the expected additional product of hiring  $L$ -th employee when illness is possible, while  $MP(\cdot)$

still denotes the additional product of hiring an additional employee with the assumption that all workers are healthy. We want to express  $MP^E(\cdot)$  in terms of  $MP(\cdot)$ , and that results from the following considerations. If all workers are sick, the  $L$ -th employee can be the only working person, or one of the two working (if all but one other employees are sick), or one of three, etc. Obviously,  $L$ -th employee has to be healthy to contribute to the overall product. The following equation follows:

$$MP^E(L) = \underbrace{(1-s)}_{(I)} \underbrace{\sum_{i=1}^L}_{(II)} \underbrace{\binom{L-1}{i-1} s^{L-i} (1-s)^{i-1}}_{(III)} \underbrace{MP(i)}_{(IV)}, \quad (4)$$

where

- (I) stands for the  $L$ -th hired worker contributing only when healthy,
- (II) index  $i$  goes over all possibilities of the  $L$ -th hired worker being exactly the  $i$ -th worker reporting for work (i.e.,  $i-1$  other workers are healthy),
- (III) measures the probability of exactly  $i-1$  other workers being healthy,
- (IV) denotes the  $L$ -th hired worker's (as the  $i$ -th working one) contribution.

Equation 4 results in  $MP^E(\cdot)$  being—as compared with  $MP(\cdot)$ —scaled downwards by (I) and stretched rightwards, as the values of  $MP^E(L)$  are determined by the values of  $MP(i)$  for all  $i \leq L$ . These relations between  $MP$  and  $MP^E$  are illustrated, e.g., in Figure 1. In this particular case the optimal level of employment does not change, and so  $L_h^* = L_s^*$ .

It can be shown that introducing the risk of illnesses does not change the general properties of the marginal productivity, i.e.,  $MP^E(\cdot)$  behaves qualitatively in the same way as  $MP(\cdot)$  does. That is expressed more formally by the following two propositions.

**Proposition 1** *Assume  $MP(L)$  to be decreasing in  $L$ , then  $MP^E(L)$  as defined in equation 4 is decreasing in  $L$  for any  $s$ .*

Hence, diminishing marginal productivity is preserved under sickness in the model. The analogous result holds for the teamwork case (with initially increasing marginal productivity).

**Proposition 2** *Assume  $MP(L)$  to be first increasing, up to some  $L'$ , and then decreasing in  $L$ . Then  $MP^E(L)$  as defined in equation 4 is also first increasing and then decreasing in  $L$  for any  $s$ .  $MP^E(L)$  attains maximum for some value  $L'' \geq L'$ .*

The above propositions mean that after introducing illness in our model we can still be looking for the optimal number of workers by looking at the point where  $MP^E(\cdot)$  has dropped below  $\frac{w}{p}$  for the first time (the marginal productivity is not multimodal). For a decreasing  $MP(\cdot)$  also  $MP^E(\cdot)$  is decreasing and so no general statements can be made as to where it will drop below  $\frac{w}{p}$ . For an initially-increasing  $MP(\cdot)$ , however,  $MP^E(\cdot)$  is also initially increasing

and attains maximum for a greater or equal number of workers than  $MP(\cdot)$  does. As the following proposition says, this maximum increases with the risk of illness.

**Proposition 3** *Assume  $MP(L)$  to be first increasing, up to some  $L'$ , and then decreasing in  $L$ . Let*

$$L''(s) = \max \left( \underset{L \in \{0,1,\dots\}}{\operatorname{argmax}} (MP^E(L)) \right)$$

for a given risk of illness,  $s$ . Then  $L''(s)$  is non-decreasing in  $s$ .

The proposition is formulated in a bit complicated a way so as to avoid comparing sets when several values maximize  $MP^E(\cdot)$  and to remain true when  $s = 1$  (and  $MP^E(\cdot) \equiv 0$ ). Surely, the optimal number of workers must be greater than or equal to the largest of the  $\operatorname{argmax}$  (irrespective of wage, but assuming the company wants to hire at all), and so this lower bound for optimal workforce increases with the risk of illness for teamwork-oriented marginal productivity. In this sense the teamwork makes it more plausible that the (foreseen) risk of illness will lead to an increased employment and so the reduced product loss due to absenteeism.

### 2.3 Output guarantees and shortfall penalties

As mentioned in the Introduction, the impact of product shortfall penalties is subject to research (e.g., [Pauly et al, 2002](#)). So as to verify how it blends with companies adjusting to the possible sick leaves, we include this element in our model in examples IV–VI (section 3.2).

We assume the market requires the company to predefine the amount of goods delivered. The company may freely choose this amount, but it needs to sign a contract before knowing the actual number of workers showing up for work in a given period. The consumers will purchase exactly the contracted amount goods (or what's available, if less). We keep the assumption that the good market is perfectly competitive and that the company is a price-taker. If the company falls short of the promised delivery, it needs (additionally to not collecting any revenue) to pay a penalty for the shortfall. For simplicity we assume that the penalty is equal to the price  $p$ , but obviously other values could be used. We assume that the amount of contracted output does not influence the price.

A possible interpretation is that the potential customers want to be assured of receiving goods from particular manufacturer and require in advance contracts promising a particular size of order. Obviously, a mixed model might also be considered: some companies promising delivery and paying penalties otherwise but charging higher price, and others *ad hoc* filling in the gaps (the price depending on the amount of gaps via some market equilibrium setting mechanism). A single company might actually combine these two schemes, i.e.,

sell the produced goods in two schemes. Not to overly complicate the model and its interpretation, we assume that all the companies have to predefine their output.

The expected profit with guaranteed output (denoted by a superscript  $G$ ) is now given by

$$\pi^{EG}(L) = p \sum_{i=0}^L \binom{L}{i} s^{L-i} (1-s)^i \underbrace{(Y(i) - |G - Y(i)|)}_{(I)} - wL, \quad (5)$$

where (I) denotes the effectively charged product (notice it is equal to  $G$  if  $Y(i) \geq G$ , and when  $Y(i) < G$  the company has to pay a penalty). The company maximizes equation 5 selecting the level of employment ( $L$ ) and the guaranteed delivery ( $G$ ). In what follows we find numerically the optimal values of these parameters on a grid, and use the same notation, i.e.,  $L_h^*$  and  $L_s^*$  ( $G_h^*$  and  $G_s^*$ ) for the case without and with sick leaves, respectively. We consider natural values of  $G$ , only. We keep on assuming company's risk neutrality.

### 3 Examples

In this section we present and discuss six scenarios how the illness impacts the companies' functioning: examples I–III (Section 3.1) without and examples IV–VI (Section 3.2) with guaranteed output/penalties. The companies differ between the examples with respect to the nature of the marginal productivity function (and so the teamwork orientation, as discussed above in Section 2.1). What is important are the qualitative differences between the examples, but so as to numerically obtain them we have to assign specific values to  $Y(\cdot)$ ,  $w$ ,  $p$ , and  $s$ . These values are, however, of no special importance. For brevity we omit the presentation of  $Y(\cdot)$  altogether, as it would require specifying multiple numbers and the general shape can be read off the figures.

In each example we analyse the impact of the possibility of illness, i.e., the difference between the outcomes for a given  $s$  and when forcing  $s = 0$ , i.e., for  $L_s^*$  and  $L_h^*$ , respectively. We focus on the impact of illness on the total product delivered to the market, i.e., indirect cost, and not on the profit.

Apart from illustrating possible, qualitatively-different situations, when presenting the examples we are able to disentangle and discuss various effects and mechanisms causing IC, that we feel to be a contribution in itself, that can be subsequently used in analysing altogether different models.

#### 3.1 No guaranteed output

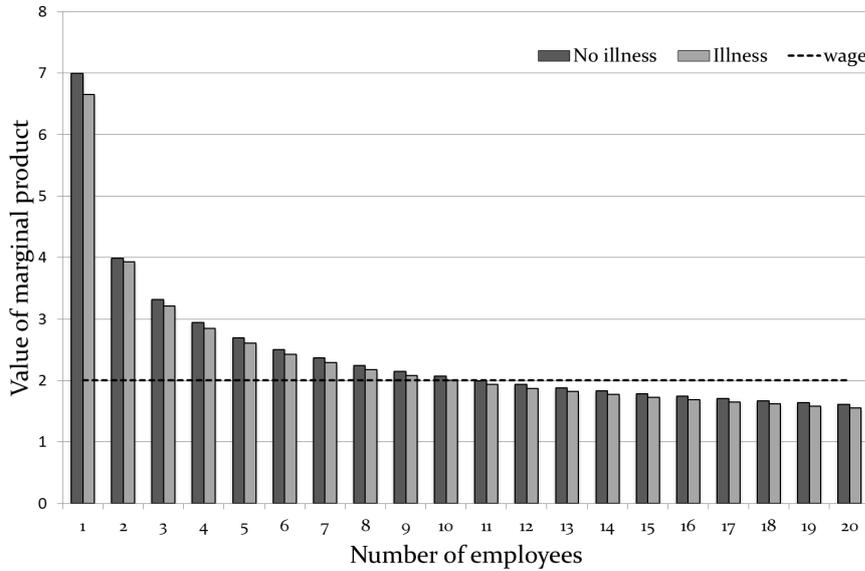
##### 3.1.1 Example I

Take a company characterised by a declining marginal productivity (Figure 1, dark bars), a small probability of illness ( $s = 5\%$ ),  $w = 2$ , and a normalized

price  $p = 1$ . We can see that  $MP^E(L) < MP(L)$  (according to Proposition 1), and so the possibility of an illness can only result in hiring the same or lower number of workers for any wage. With an assumed  $w = 2$  it is optimal to hire ten employees, both if there were no illness and with  $s = 5\%$ , i.e.,  $L_h^* = L_s^* = 10$ . Thus, even with expectations there are no adjustments. Still, the possibility of illness occurrence leads to  $Y^E(L) < Y(L)$ , and so in particular  $Y^E(10) < Y(10)$ : the same number of employees delivers (in terms of expected values) a product smaller by  $Y(10) - Y^E(10) \approx 1.044$  (graphically in Figure 1 it is the difference in the area of the first ten dark and light bars). As  $p = 1$ , this is exactly the value of a lost product, hence IC.

In applied research IC is usually estimated as the number of sick leaves multiplied by wage, and so in our case  $L_s^* \times s \times w = 10 \times 5\% \times 2 = 1$ . The difference between the actual and approximated IC is small.

Notice that here IC results only from the actual occurrence of illness, and is neither alleviated nor increased by company's adjustments. As mentioned, we assumed that the adjustments do not change the equilibrium price. One might argue that all the companies in the market are subject to the possibility of illness, and so all will reduce the delivered output. Then the overall supply curve will move left, and the equilibrium price should increase. We should then perhaps define IC to be  $Y(L_h^*)p - Y^E(L_s^*)p'$ , where  $p'$  denotes the new equilibrium price resulting from a slightly reduced output. As  $p' > p$ , IC is now estimated to be lower. We come back to this issue in the last section, after presenting the remaining examples.



**Fig. 1** Marginal productivity of labour in Example I (with and without illness).

### 3.1.2 Example II

We consider a greater probability of illness,  $s = 10\%$ , and we slightly modify wage to  $w = 2.5$ . Should there be no illness it would be optimal to hire nine employees,  $L_h^* = 9$ , but with illness it drops to  $L_s^* = 7$ , as illustrated in Figure 2. Analogously to Example I, the expected indirect cost of illness is thus given by  $(Y(L_h^*) - Y^E(L_s^*)) \times p = (Y(9) - Y^E(7)) \times 1$ , in Figure 2 the difference in area between the first nine dark bars and the seven light bars. Notice that the expression for indirect cost can be rewritten in the following form

$$\begin{aligned}
 & \left( \underbrace{Y(L_h^*) - Y(L_s^*)}_{\text{(I)}} + \underbrace{Y(L_s^*) - Y^E(L_s^*)}_{\text{(II)}} \right) \times p = \\
 & = \left( \underbrace{Y(9) - Y(7)}_{\text{(I)}} + \underbrace{Y(7) - Y^E(7)}_{\text{(II)}} \right) \times 1 = \\
 & = (5.257 + 1.989) \times 1 = 7.246,
 \end{aligned} \tag{6}$$

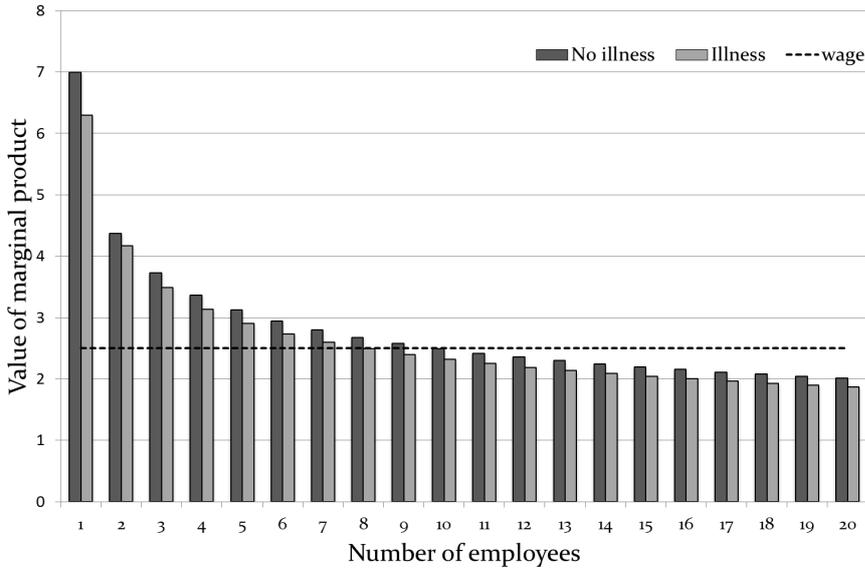
with both (I) and (II) positive (and both easily readable off Figure 2: the difference between nine or seven dark bars vs the difference between seven dark or seven light bars, respectively). Then (I) denotes the loss in the product resulting from the company adjusting to the expected sick-leaves and effectively reduced marginal productivity of consecutive employees, and (II) denotes the loss resulting from the actual sick-leaves in the prepared company. As easily seen in Figure 2, (I) can be much larger than (II). Thus, it is the expectations and adjustments that are responsible for the major part of indirect cost.

Applying the standard rules of calculating indirect cost would yield  $L_s^*sw$ , i.e.,  $7 \times 10\% \times 2.5 = 1.75$ , and so much less than the actual loss  $(Y(L_h^*) - Y^E(L_s^*)) \times p = 7.246$ . That shows how much we may underestimate IC if we only base our estimates on the observed number of sick-leaves, not accounting for the impact of the general phenomenon of illness occurrence on the company's hiring policy.

It is worth to mention that the difference between parameters in this and the previous example is only quantitative, i.e., in both examples we assumed a declining MP. That means that only quantitative changes in input parameters can cause qualitative differences in the results concerning the value of IC, as well as the main mechanism responsible for IC generation.

### 3.1.3 Example III

In Examples I&II we assumed a declining marginal productivity of labour. We now assume that marginal productivity first increases only then to be diminishing, as illustrated in Figure 3. That means that hiring additional employees increases the total product up to some particular threshold, and beyond it hiring an additional worker offers only a decreasing marginal product. In our



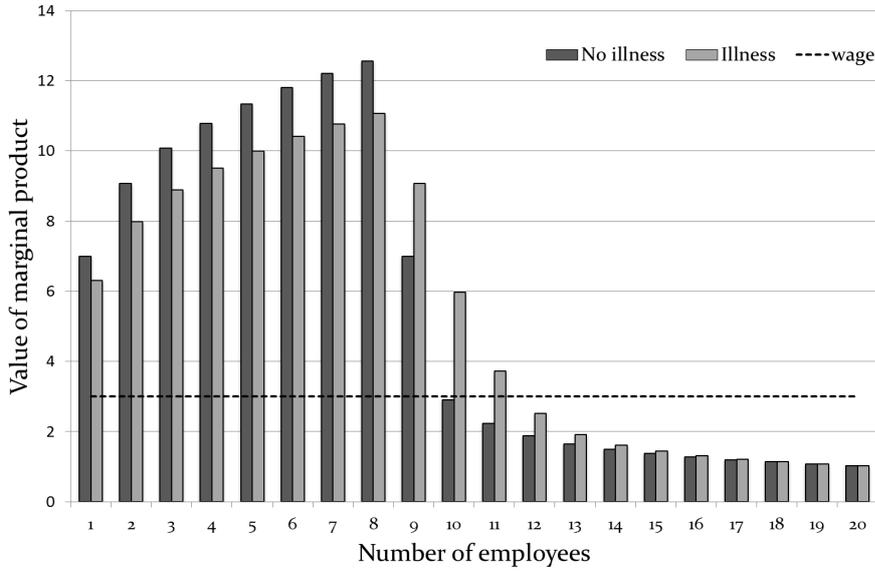
**Fig. 2** Marginal productivity of labour in Example II (with and without illness).

particular case  $MP(\cdot)$  increases up to eight employees, hiring the ninth, tenth, ... employee offers smaller and smaller gains in the total product. This type of situation can occur in companies that offer a non-scalable good (e.g., have to deliver a complete project) requiring a team of some size, but also have some defined capacity limiting the productivity when the number of employees is increased.

We take  $w = 3$  and the probability of a sickness  $s = 10\%$ , as in Example II. In this case  $L_h^* = 9$  but, surprisingly, the optimal employment is greater accounting for illness:  $L_s^* = 11$ . That is the result of the marginal productivity defined in equation 4 being smeared rightwards as discussed in subsection 2.2. Notice that optimal employment will be lower or equal to the optimal employment in case of illness for any  $s$  (Proposition 2). The indirect cost is now given as (analogously to equation 6)

$$\begin{aligned} & \left( \underbrace{Y(9) - Y(11)}_{(I)} + \underbrace{Y(11) - Y^E(11)}_{(II)} \right) \times p = \\ & = (-5.124 + 3.301) \times 1 = -1.823. \end{aligned} \quad (7)$$

Now (I) is negative: company's adjustments lead to reduction in IC, and in sum we have negative IC (we might call it *indirect gains*). Estimating the indirect cost as  $L_s^*sw = 3.3$  would obviously lead to overestimation. In general the net effect can be both negative and positive, but as Example III shows it is possible that illnesses will result (when accounting for companies preparing for them) in net gain in the value of product delivered to the market.



**Fig. 3** Marginal productivity of labour in Example III (with and without illness).

### 3.2 Guaranteed output

#### 3.2.1 Example IV

We take the marginal product function as presented in Figure 4,  $p = 1$ , and  $w = 4.9$ . Without illness, the company would hire six employees and contract the whole output, i.e.,  $L_h^* = 6$ ,  $Y(L_h^*) = 45$  (the sum of the first six dark bars in the figure), and  $G_h^* = 45$ .

We consider the risk of a sick-leave  $s = 25\%$ . Looking numerically for the optimal combination of  $L$  and  $G$  we obtain  $L_s^* = 5$  and  $G_s^* = 34$ . Notice that, again, there are two sources for indirect cost: company reducing the number of hired workers due to foreseeing the possibility of them being ill and workers actually getting ill, thus reducing the product. In the present example having to contract the output beforehand does not change the number of workers hired: the company would also hire five workers simply by looking at the  $MP^E(\cdot)$ . In this case there is an additional effect, however. Notice that  $Y(5) > 34$ , which means that it may happen (with probability  $75\%^5 \approx 24\%$ ) that all five workers will be available but still, due to (optimally set)  $G_s^*$  the company will only produce 34 units of good.

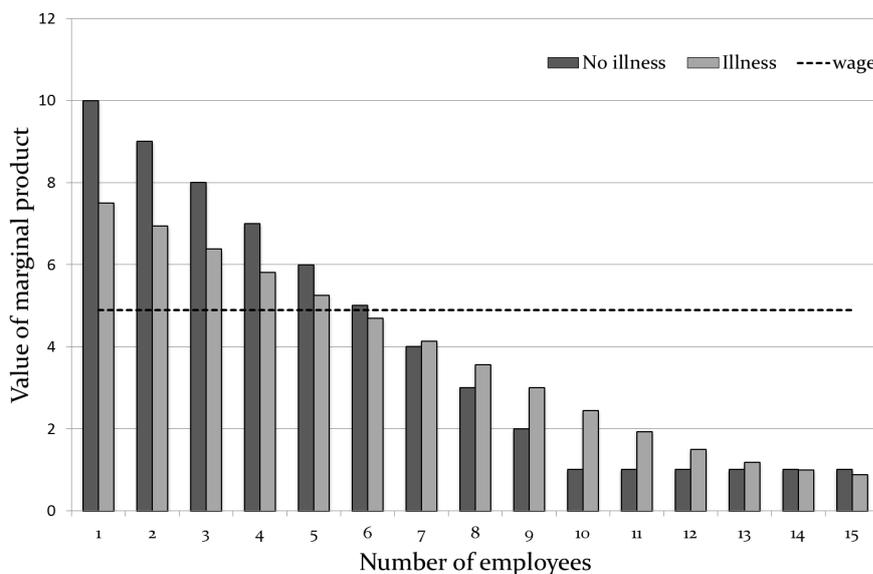
Whether this  $G_s^* < Y(L_s^*)$  should be treated as a constituent of IC is disputable in our view. On one hand, it deprives the market of the goods that could be delivered. On the other hand, the consumers beforehand made contracts with companies to obtain required goods, and so there are no consumers *expecting* the company under consideration to deliver  $>34$  units. Broadening

the picture,  $G_s^* < Y(L_s^*)$  can be treated as IC due to the following reason. Companies being willing only to guarantee a limited amount of good (and not using full production capacity in cases when all workers are healthy) reduces the average output. That aggregates over the entire market and moves the supply curve to the left, then the very equilibrium point is shifted left and up, and less good is delivered (at a higher equilibrium price). This reduction in the equilibrium amount can be treated as IC. The increase in equilibrium price may lead to new entries under our assumption of a perfect competition, reducing the effect, but the contracting may be happening not long before the delivery, and so disappointingly low contracts may not provide enough time for new entries.

Summing up, with no illness the company would hire six workers and promise (and deliver) 45 units of good. Expectations and adjustments result in hiring five workers only and promising on the safe side 34 units of good. Thus the expectations and adjustments, even with no illness actually occurring lead to  $IC = 11$ . Workers actually getting ill reduce the expected delivered product further to ca. 30.45, increasing IC to ca. 14.55.

Should there be no contracting, the company would still hire five workers but would not artificially limit the production to  $G_s^* = 34$ . Then the expected delivered output would amount to 31.875 units. Thus, guaranteeing output and shortfall penalties *per se* contributes to the indirect cost.

Taking the usual approach yields  $L_s^*sw = 6.125$  and greatly underestimates the value of IC.



**Fig. 4** Marginal productivity of labour in Example IV (with and without illness).

### 3.2.2 Example V

We modify, comparing with example IV,  $p = 2$  and  $w = 4.9$ . The  $MP(\cdot)$  function is the same as in Example IV (section 3.2.1). With no illness it would be optimal to hire eight workers,  $L_h^* = 8$ , and contract&deliver  $Y(L_h^*) = 52$ . If we take a high risk of illness,  $s = 40\%$ , then  $L_s^* = 12$  even though  $MP^E(12) < w$  (see Figure 5). It results from the fact that hiring many employees allows to manage the delivery of a contracted output better. The company will set  $G^* = 49$ , would be able to produce 48.26 units on average, but will actually produce 46.39 units on average, only (not using its full capacity on some occasions, due to limited guaranteed output). IC is thus equal to 5.61.

Interestingly, now the guaranteed output mechanism reduces IC: without it  $L_s^* = 10$  (looking at the  $MP^E(\cdot)$  function, see Figure 5), and deliver 43.8 on average.

Taking the regular approach yields  $L_s^*sw = 12 \times 40\% \times 4.9 = 23.52$ , and so greatly overestimates IC.

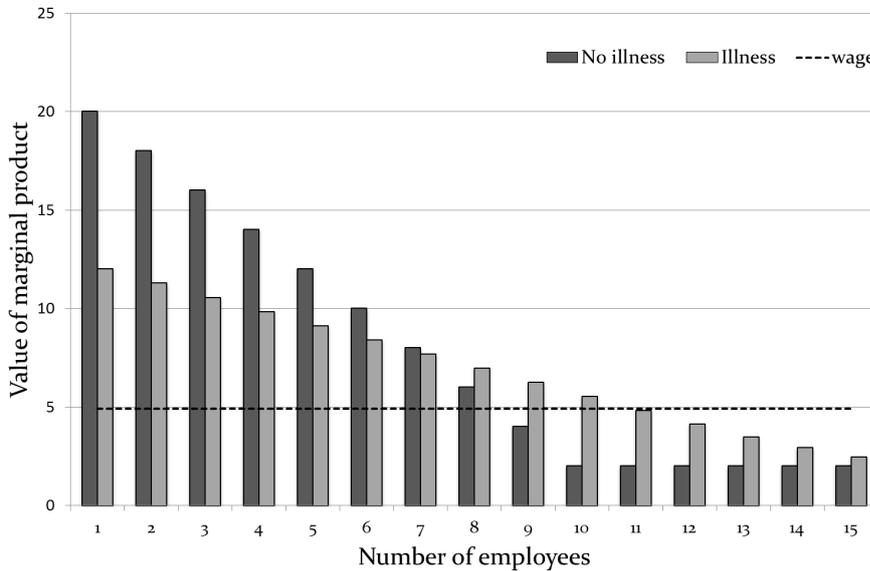


Fig. 5 Marginal productivity of labour in Example V (with and without illness).

### 3.2.3 Example VI

Continuing the analysis from Example V, it turns out that it is possible that introducing illness can lead to the increase in both employment and guaranteed output. Take the marginal product function as presented in Figure 6. We assume that  $p = 3$  and  $w = 4.9$ . Without illness, the company will hire  $L_h^* = 9$

workers and will produce  $Y(L_h^*) = 54$  units of good. By adopting  $s = 5\%$  we obtain the optimal employment of  $L_s^* = 10$  workers and  $G_s^* = 55$  (the total potential output). The actual (and potential) expected level of product is equal to 54.39 units. Just as in Example V guaranteeing output and shortfall penalty lead to increase of employment and reduce IC (here turning it into indirect gains).

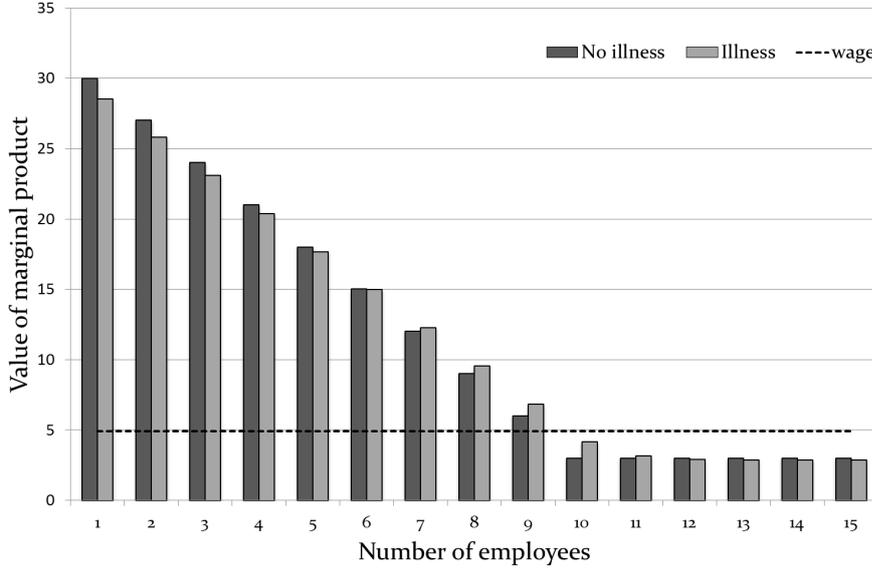


Fig. 6 Marginal productivity of labour in Example VI (with and without illness).

#### 4 Discussion

We showed in the previous section that sick leaves combined with company expecting them and preparing for them may have different impact on company's functioning and IC. Companies aware of the risk of the sick leaves may both hire fewer workers or hire more of them. In the former case the actual IC is greater than as estimated using standard methods (multiplying observed average sick leaves by the daily wage), in the latter case the IC is reduced and even possibly turned into indirect gains. In our model the former case is associated with diminishing marginal productivity, and the latter is more easily associated with an initially-increasing marginal productivity (which we associate with teamwork). As shown by Examples V&VI having to pay shortfall penalties for undelivered guaranteed output may actually reduce IC.

The above findings are contrary to what was observed by Pauly et al (2002). The difference lies in our approach to finding substitutes for sick workers.

Pauly et al's result hold when the cost of finding substitutes exceeds the wage. In our case we do not model the process of looking for substitutes but the possibility to hire more workers preparing for the sick leaves ( $L_s^* > L_h^*$  as in Examples III,V&VI) can be in a sense treated as hiring substitutes for the expected sick workers at the standard wage  $w$ . Their and our results are not in disagreement under this interpretation.

Our model allows to differentiate between various mechanisms generating IC, some of them not included in the standard ( $L^*sw$ ) approach. In general, we believe that when analysing IC more attention should be paid to how illness as a phenomenon (as opposed to the actual occurrence) distorts the companies' functioning and generates cost to the society (we are not interested in cost to companies when discussing IC). As Bryson (2013) notices, companies might use temporary agency workers (TAW) to fill in for short illness-related absence. The impact of using TAWs (on the company, on regular workers hired in that company, on TAWs themselves) also constitutes part of the indirect cost of illness. We might also go beyond the hiring or contracting policy. For example in the present study the firm cannot store its product. Storing products would smooth the impact of shocks caused by illnesses, and looks especially attractive when the company faces shortfall penalties. However, it should be noted that storing goods generates cost (e.g., froze capital, insurance, storage area, guarding). This cost should be added as a part of IC (as it is the opportunity cost as well), and so it is difficult to assess the net impact on IC.

We have to agree that our approach to modelling teamwork is a bit naïve. We do not, for example, directly model the variety of competences needed in a team to complete the project, which may be perceived as a definition of teamwork. In real life the workers in the same company may differ with respect to how team-work-oriented their job is (e.g., Pauly et al, 2008, estimated the multipliers measuring the effect of worker's absence on the product for various professions). Accounting for this would require defining various types of labour and extending the model, which is left for further research.

It can of course be challenged whether firms apply economic models in business practice to optimize their processes. Probably, a large number of entrepreneurs running small and medium-size firms do not know the definition of  $MP(L)$ . However, if we assume that firms aim to maximise profits, then companies by trial-and-error (e.g., analysing profits generated by varying employment levels caused by the natural rotation) can find optimal values of parameters. Moreover, firms applying optimal solutions will obtain better results, and assuming such a behaviour is rewarded (e.g., by greater resistance to shocks and a smaller risk of bankruptcy), should prevail in the market. Alternatively, the present study can be treated as an analysis of what will be happening in ideal, optimally managed company.

We assumed that the employer pays the ill, which may not be true for all the markets. This assumption generally holds, e.g., in Poland, where the employer pays the salary for the first 33 days of the illness (per calendar year, with some exceptions depending on the age of the employee), and then the public insurer takes over (in both cases the employee gets only 80% of a

regular wage). Similar model is used in Germany, where the employer pays full salary for at least first 6 weeks of illness, then the insurer pays 70% of salary. In Sweden, employer covers sick-leaves for the first two weeks of illness. Should the salaries be paid by the insurer during the whole illness, that would probably result in wages being greater so as to include premium paid to the insurer, which would not change the expected values and the results. That might be formally verified in further research.

We did not aim to opt for HCA or FCM method, but our analysis suits the latter more. We neglected the possibility of replacements, what might look as using an HCA approach, but that results mostly from analysing a short time period. Importantly, we limited our analysis to the functioning of the single firm and its employees. Reducing the number of employees, and hence the output, was considered as IC. That is much closer to FCM, because we look at the actual impact on delivered goods. In HCA we focus on what happens to the capital impersonated—the people are simply not hired by one company, and not ill, thus the capital is available and no IC is generated (in a sense: we assume full employment in the market, and so these people will simply be hired by some other company).

Perhaps, then, a wider view is needed, i.e., we should consider how changes in single companies' policy will affect the labour and good markets as a whole. No qualitative changes are to be expected: for example reducing the demand for labour will move the demand curve to the left by some amount. The new equilibrium price (i.e., wage) will fall, reducing the impact on the equilibrium quantity (number of hired workers), but we still expect fewer employees. Thus, widening the scope to the whole market most likely will reduce the scale of the effect, not changing its direction. It may be more complicated when we account for the fact that the same illness may influence companies varying with respect to the marginal productivity. Then some companies may be hiring more, and some may be hiring less, and these two effects may cancel each other out to some extent.

Finally, we might (provocatively perhaps) ask whether the changes in equilibrium price of a good should be accounted for when estimating IC. For example, an increase of a good price results in a greater value of delivered goods, artificially generating indirect gains, while the society does not seem to be better off. Notice however, that the increase in price will (in our considerations focused on the supply side) result from moving the supply curve to the left, thus will be associated with a decrease in the amount (and so IC). Including the changes in price and quantity means comparing  $y'p' - yp$ , where prime denotes values after the change in equilibrium ( $y' < y$ ,  $p' > p$ ). The increase in price then alleviates the impact of the decrease in quantity and can be motivated by the following observation: the actual decrease in quantity results in the good not being delivered to (purchased by) persons who valued this good least (and are not willing to pay  $p' > p$ ). IC is then downwards corrected. We do not pursue this issue further, as it seems to require dedicated considerations—the more general market mechanisms we want to consider, the more dilemmas we have to resolve what should be counted as IC constituent.

## 5 Conclusions

We presented a model of a company allowing to introduce the risk of illness and firm's adjustments in its hiring policy and contracting policy (guaranteed output). We showed that the risk of illness does not change the qualitative properties of the model, in a sense of a shape of a marginal productivity function (propositions 1–3). Teamwork (where marginal productivity is initially increasing) more easily may lead to an increased employment in expectation of the risk of illness, thus in our model teamwork results in companies preparing for sick-leaves in a way that reduces the indirect cost (see proposition 2&3 & example III)—a new insight as compared to the literature. As our examples show, a variety of situations are possible when it comes to the actual, quantitative impact of illness on IC with adjusting companies: IC may be increased, or reduced, even up to becoming negative (examples III&VI). As Example VI shows, having to guarantee output beforehand under penalty may also lead to negative IC.

This variety of outcomes shows that few general results can be proved. Apart for propositions 1–3, we would like to draw another general conclusion, however. Illnesses have dual impact on the firms' functioning: the possibility of illness occurring impacts how firms organise their processes; the actual occurrence impacts the ultimate results. Hence, when calculating IC two streams should be explicitly accounted for: the impact of companies adjustments and the impact of the actual occurrence of the illness. Methods typically presented in the literature do not look at the former and so may yield biased results. As we show, the interplay between the two mechanisms can result in illnesses generating indirect cost much different (in both directions) than as calculated with typically used formulas. Considering various market (labour and good, possibly also capital) mechanisms is required to fully understand how indirect cost should be measured.

## Appendix

*Proof (Proof of Proposition 1)* We first show how  $MP^E(\cdot)$  can be presented in a different form, more convenient for the present proof. We start with rewriting equation 4 and then modify it (notice we write it for  $(L + 1)$ -th worker here

and change how the summation range is denoted).

$$\begin{aligned}
\frac{\text{MP}^E(L+1)}{1-s} &= \sum_{i=0}^L \binom{L}{i} s^{L-i} (1-s)^i \text{MP}(i+1) = \\
&= \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-i} (1-s)^i \text{MP}(i+1) + \\
&\quad + \sum_{i=1}^L \binom{L-1}{i-1} s^{L-i} (1-s)^i \text{MP}(i+1) = \\
&= s \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \text{MP}(i+1) + \\
&\quad + (1-s) \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \text{MP}(i+2) = \\
&= s \times \frac{\text{MP}^E(L)}{1-s} + (1-s) \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \text{MP}(i+2),
\end{aligned} \tag{8}$$

where we use the fact that for  $i \in \mathbf{N}$ ,  $0 < i < L$ ,  $\binom{L}{i} = \binom{L-1}{i} + \binom{L-1}{i-1}$ . Hence,  $\text{MP}^E(L+1)$  is a weighted average (with weights  $s$  and  $1-s$ ) of  $\text{MP}^E(L)$  and a similarly calculated expression in which we take MPs moved one worker to the right. As  $\text{MP}(\cdot)$  is decreasing it must be that

$$\sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \text{MP}(i+1) \geq \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \text{MP}(i+2),$$

as  $\geq$  holds for each term of the summation.

*Proof (Proof of Proposition 2)* The proof takes several steps. First, notice that  $\text{MP}^E(1) = (1-s)\text{MP}(1)$ , and  $\text{MP}^E(2) = (1-s)(s \times \text{MP}(1) + (1-s) \times \text{MP}(2))$ . Then, if  $\text{MP}(2) \geq \text{MP}(1)$ , then  $\text{MP}^E(2) \geq \text{MP}^E(1)$  for any  $s$ . Thus, if  $\text{MP}(\cdot)$  is increasing initially, so is  $\text{MP}^E(\cdot)$ .

Notice, that if  $\text{MP}(\cdot)$  is decreasing after some point, then  $\text{MP}^E$  must also decrease at some point. Say, we calculate  $\text{MP}^E(\cdot)$  for some  $0 \ll L_1 \ll L_2$ , where already  $L_1$  is much greater than the point where  $\text{MP}(\cdot)$  started to be decreasing. Increasing  $L_2$  results in  $\text{MP}^E(L_2)$  being calculated (as a weighted sum in equation 4) averaging lower values of  $\text{MP}(\cdot)$  than when calculating  $\text{MP}^E(L_1)$  (the weights assigned to the values of  $\text{MP}(\cdot)$  prevailing in both sums can be made arbitrarily small by increasing  $L_2$ ).

We now show that if  $\text{MP}^E(L+1) < \text{MP}^E(L)$  then  $\text{MP}^E(\cdot)$  must be strictly decreasing further on. This is the main part of the whole proof, and again it is done in several steps. First,  $\text{MP}^E(\cdot)$  is calculated as an expected value of  $\text{MP}(\cdot)$  function calculated for a variable distributed according to a binomial distribution. It is then useful to see how the probabilities change in this distribution. Denote by  $B(k, n, p)$  the probability of  $k$  successes in  $n$  independent

experiments, where a single success comes with probability  $p$ . We are interested in

$$\frac{B(k, n, p)}{B(k+1, n, p)} = \frac{\binom{n}{k} p^k (1-p)^{n-k}}{\binom{n}{k+1} p^{k+1} (1-p)^{n-k-1}} = \frac{k+1}{n-k} \times \frac{1-p}{p}, \quad (9)$$

and this is increasing in  $k$ .

As noticed in the proof of Proposition 1,  $MP^E(L+1)$  is calculated as a weighted average of  $MP^E(L)$  (calculated based on  $MP(\cdot)$  for workers from 1 to  $L$ ) and a weighted average of  $MP$  calculated for workers from 2 to  $(L+1)$  with weights given by a binomial distribution  $B(\cdot, L-1, 1-s)$ . It will be useful to show that if

$$\sum_{i=0}^{L-1} B(i, L-1, 1-s) MP(i+1) > \sum_{i=0}^{L-1} B(i, L-1, 1-s) MP(i+2), \quad (10)$$

and so  $MP^E(L+1) < MP^E(L)$ , then also

$$\sum_{i=0}^{L-1} B(i, L-1, 1-s) MP(i+2) > \sum_{i=0}^{L-1} B(i, L-1, 1-s) MP(i+3). \quad (11)$$

Let  $\Delta(i)$  denote  $MP(i) - MP(i-1)$ , defined for  $i \geq 2$ .  $\Delta(\cdot)$  is positive for the part of  $MP(\cdot)$  with increasing marginal productivity. Our assumptions guarantee that  $\Delta(\cdot)$  changes from positive to negative. Rewriting inequality 10

$$\sum_{i=0}^{L-1} B(i, L-1, 1-s) \Delta(i+2) < 0, \quad (12)$$

and, analogously, in order to prove inequality 11 we need to show that

$$\sum_{i=0}^{L-1} B(i, L-1, 1-s) \Delta(i+3) < 0. \quad (13)$$

Obviously some  $\Delta(\cdot)$  values in inequality 12 must be negative. Assume non-trivially that also some (for  $i \leq i^*$ ) are positive. Then

$$\begin{aligned} \sum_{i=0}^{L-1} B(i, L-1, 1-s) \Delta(i+2) &\geq \sum_{i=1}^{L-1} B(i, L-1, 1-s) \Delta(i+2) = \\ &= \underbrace{\sum_{i=1}^{i^*} B(i, L-1, 1-s) \Delta(i+2)}_{\text{(I)}} + \underbrace{\sum_{i=i^*+1}^{L-1} B(i, L-1, 1-s) \Delta(i+2)}_{\text{(II)}}, \end{aligned}$$

where (I) is positive, and (II) is negative. We have (I) + (II) < 0, and so

$$\sum_{i=1}^{i^*} B(i-1, L-1, 1-s) \Delta(i+2) + \sum_{i=i^*+1}^{L-1} B(i-1, L-1, 1-s) \Delta(i+2) < 0,$$

as equation 9 is increasing in  $k$ , and so the negative elements in (II) are inflated more than the positive elements in (I). Rewriting the last inequality we get

$$\begin{aligned} \sum_{i=1}^{L-1} B(i-1, L-1, 1-s)\Delta(i+2) &= \sum_{i=0}^{L-2} B(i, L-1, 1-s)\Delta(i+3) \geq \\ &\geq \sum_{i=0}^{L-1} B(i, L-1, 1-s)\Delta(i+3), \end{aligned}$$

which proves inequality 13.

As the final step notice that just as we decomposed  $\text{MP}^E(L+1)$ , we can decompose  $\text{MP}^E(L+2)$  as follows

$$\begin{aligned} \frac{\text{MP}^E(L+2)}{1-s} &= s^2 \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+1) + \\ &+ 2s(1-s) \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+2) + \\ &+ (1-s)^2 \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+3) = \\ &= s^2 \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+1) + \\ &+ 2s(1-s) \sum_{i=0}^{L-1} B(i, L-1, 1-s)(\text{MP}(i+1) + \Delta(i+2)) + \\ &+ (1-s)^2 \sum_{i=0}^{L-1} B(i, L-1, 1-s)(\text{MP}(i+1) + \Delta(i+2) + \Delta(i+3)) = \\ &= \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+1) + \\ &+ (1-s^2) \sum_{i=0}^{L-1} B(i, L-1, 1-s)\Delta(i+2) + \\ &+ (1-s)^2 \sum_{i=0}^{L-1} B(i, L-1, 1-s)\Delta(i+3) < \\ &< \sum_{i=0}^{L-1} B(i, L-1, 1-s)\text{MP}(i+1) + \\ &+ (1-s) \sum_{i=0}^{L-1} B(i, L-1, 1-s)\Delta(i+2) = \\ &= \frac{\text{MP}^E(L+1)}{1-s}, \end{aligned} \tag{14}$$

where the inequality results from omitting a negative term and  $1 - s^2 \geq 1 - s$ . This finishes the proof of  $\text{MP}^E(\cdot)$  being decreasing ever since the first decrease.

Finally, to show that the first decrease will only happen for some  $L$  greater than the point at which  $\text{MP}(\cdot)$  attains its maximum, it's enough to revert the argument presented in the proof of proposition 1.

*Proof (Proof of Proposition 3)* We will show that if for some  $L$  we have  $\text{MP}^E(L) \leq \text{MP}^E(L+1) \geq \text{MP}^E(L+2)$ , then the difference  $\text{MP}^E(L+2) - \text{MP}^E(L+1)$  is strictly increasing in  $s$ . Based on the decomposition presented in the proof of Proposition 1, this difference amounts to

$$s \times \text{MP}^E(L+1) + (1-s)^2 \sum_{i=0}^L \binom{L}{i} (1-s)^i s^{L-i} \text{MP}(i+2) - \text{MP}^E(L+1),$$

and so to

$$(1-s)^2 \sum_{i=0}^L \binom{L}{i} (1-s)^i s^{L-i} \text{MP}(i+2) - (1-s)^2 \sum_{i=0}^L \binom{L}{i} (1-s)^i s^{L-i} \text{MP}(i+1)$$

or, more concisely, to  $(1-s)^2 \sum_{i=0}^L \binom{L}{i} (1-s)^i s^{L-i} \Delta(i+2)$ .

The derivative of this expression with respect to  $s$  is given as (I) + (II) + (III) + (IV) + (V), where

$$(I) = -2(1-s) \sum_{i=0}^L \binom{L}{i} (1-s)^i s^{L-i} \Delta(i+2),$$

$$(II) = (1-s)^2 \binom{L}{0} L s^{L-1} \Delta(2),$$

$$(III) = -(1-s)^2 \sum_{i=1}^{L-1} \frac{L!}{(i-1)!(L-i)!} (1-s)^{i-1} s^{L-i} \Delta(i+2),$$

$$(IV) = (1-s)^2 \sum_{i=1}^{L-1} \frac{L!}{i!(L-i-1)!} (1-s)^i s^{L-i-1} \Delta(i+2),$$

$$(V) = -(1-s)^2 \binom{L}{L} L (1-s)^{L-1} \Delta(L+2).$$

Now, (II) + (IV) =  $(1-s)^2 L \sum_{i=0}^{L-1} \binom{L-1}{i} (1-s)^i s^{L-1-i} \Delta(i+2)$ , while (III) + (V) =  $-(1-s)^2 L \sum_{i=0}^{L-1} \binom{L-1}{i} (1-s)^i s^{L-1-i} \Delta(i+3)$ .

Notice (cf. equation 8 in the proof of Proposition 1) that

$$\text{MP}^E(L+1) - \text{MP}^E(L) = (1-s)^2 \sum_{i=0}^{L-1} \binom{L-1}{i} s^{L-1-i} (1-s)^i \Delta(i+2). \quad (15)$$

Accounting for our assumptions, that directly proves that (II) + (IV)  $\geq 0$  and (I)  $\geq 0$  (applying the formula to  $\text{MP}^E(L+2) - \text{MP}^E(L+1)$ ).

We can also decompose  $MP^E(L+2)$  into three elements as shown in first three lines of equation 14 in the proof or Proposition 2. Again, accounting for the fact that  $MP^E(L) \leq MP^E(L+1) \geq MP^E(L+2)$  it must be that  $(III) + (V) \geq 0$ .

If either  $MP^E(L) < MP^E(L+1)$  or  $MP^E(L+1) > MP^E(L+2)$ , then the derivative to be strictly positive (for  $s < 1$ ).

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