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Repeated moral hazard with costly self control

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# Repeated moral hazard with costly self control

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## Abstract

We consider a repeated principal-agent model, where a single agent exhibits problems of self control modelled using Gul and Pesendorfer (2001) type temptation preferences. In such a setting, for a parameterized strength of self-control, we solve for the optimal multi-period contract. Our analysis identifies a new channel of principal and agent interactions, that can be used to provide incentives, this being the reduction of agent's self control costs. In fact, the principal computes (and uses) agent's most tempting item but never finds it optimal to reduce the agent's self-control cost to zero. Presence of this new channel challenges typical results obtained in models with no-temptation on the agent's side. For example, the intrinsic motivation (resulting from costly self-control) can substitute for standard (external) incentives, and hence the moral hazard problem can be mitigated (for sufficiently high temptation parameter). Moreover, the optimal contract calls for a lower deferred part of the bonus (or consumption smoothing) than in the model with no temptations. Under limited commitment, presence of self-control also reduces agent's willingness to break or renegotiate the contract after output realization within some period, and make the optimal contract spot implementable (again for sufficiently high temptation). Impact of self-control on the cost of implementation as well as willingness to save/borrow is ambiguous, however.

**keywords:** repeated moral hazard, self-control costs, temptation, principal-agent, optimal contract

**JEL codes:** D86

## 1 Introduction and related literature

Since the seminal work of Strotz (1956) or more recently Laibson (1997), there is now an extensive literature stressing the importance of temptation and self-control problems in explaining individual behavior in economic models. When studying dynamic models with such dynamically inconsistent preferences, economist have attempted to develop various solutions methods to explain the behavioral observations that have been found in the empirical literature. For example, Strotz (1956) and Caplin and Leahy (2006) used the language of recursive decision theory

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to compute time consistent solutions. Alternatively, following the contributions of Phelps and Pollak (1968) or Peleg and Yaari (1973) economists have reverted to studying game theoretic constructions such as Markov perfect equilibria. Since the seminal contribution of Gul and Pendorfer (2001) (GP, henceforth), however, time consistent (or rational) representations of the models with temptation preferences, or preferences for commitment, have been presented. Their representation allows to solve many technical predicaments usually present, when using models with dynamically inconsistent preferences and, by doing so, allows to extend the analysis of temptation and self-control motives in the otherwise standard models.

Temptation and commitment problems are also present in contractual framework, e.g., within a company between managers and employees, where employees face various temptations for not exerting a desired effort level or delaying a task or a project. This is especially apparent in the multiperiod or repeated interactions. Moreover, various features of the contract or task / work arrangements can influence temptations and costs to avoid them. A number of empirical and experimental papers document such observations. For example, Kaur, Kremer, and Mullainathan (2010) demonstrate in a field experiment, how various workplace arrangements can mitigate self control problems, thereby raising labor productivity. Similarly, Kaur, Kremer, and Mullainathan (2015) document that (tempted) workers choose dominated contracts, which penalize low output but provide no greater reward for high output, as well as that chosen effort increases as the payday gets closer. Next, Beshears, Choi, Harris, Laibson, Madrian, and Sakong (2015) in an experiment compare and discuss how do specific contract features influence agents' preferences and hence their demand for commitment (e.g. illiquid savings accounts) contracts. Our approach follows this line of literature, but instead of experimental comparative statics, we endogenize a contract construction so that it is mostly preferred to the (monopolistic) principal, and then conduct comparative statics with parameter measuring the strength of temptation. Our model is general, hence apart from managerial compensation considerations, it may also encompass dynamic (e.g. insurance) firm-client contractual relations, as well as more recent discussion on endogenous defaults using dynamic moral hazard models, where agents (e.g. clients or borrowing countries) are tempted or have limited commitment.

Specifically, we construct a dynamic principal-agent model, but departure from standard one by allowing the agent to exhibit GP preferences. Our analysis is parsimonious, i.e. we consider a single-principal, a single-agent model with two actions and two outputs, and start the analysis for two periods. From this perspective the contribution of our paper is threefold: first we characterize the optimal dynamic contract, when the agent finds it costly to control himself from not succumbing to temptation in an otherwise standard dynamic moral hazard model (see Rogerson (1985) or Spear and Srivastava (1987)). Second, we conduct the comparative statics exercise and answer, how the strength of temptation changes the optimal wage scheme. Third, we identify how the presence of costly self control challenges typical results obtained in models with no-temptation on the agent's side. Finally, we consider some important extensions including many periods, many actions, the role of savings, limited commitment or renegotiation

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Our analysis identifies a new channel of principal and agent interactions that can be used to incentivize the agent, this being the reduction of its self control costs. Presence of this new channel implies, among others, that principal does not find it optimal to reduce agent's self-control cost to zero. Next, incentive compatibility constraints may be non binding at the optimal solution. Moreover the standard martingale property equating marginal cost of high (low) action today and future expected marginal payments does not hold in our case as deferred bonus (punishment) increases the cost of self control today. The higher the parameter measuring costly self control the larger the departure from the martingale property e.g. Presence of self-control also reduces agent's willingness to break or renegotiate the contract under limited commitment, and can make the optimal contract spot implementable.

This paper is related to Yilmaz (2013), who analyze the optimal contract in a two period principal-agent model, where agent's preferences are  $\beta - \delta$ . There are few points, when comparing our result to his paper. First, our paper offers a framework for considering contracts for the tempted agent, who finds it costly but still possible to refrain from temptations, rather than a time-inconsistent one, who always succumbs to temptation (the so called Strotz case). Our analysis embeds Strotz model as a limiting case. Second, our model allows to extend the analysis easily to many or infinite number of periods following Gul and Pesendorfer (2004) approach, rather than considering a dynamic model with time-inconsistent preferences that causes technical problems even with the solution to the agent maximization problem<sup>1</sup>. Third, many contributions following GP framework allow to consider and parameterize various aspects of self-control, including Dekel, Lipman, and Rustichini (2009) random temptations, Olszewski (2011) choice dependent temptations or dynamic temptations (see Noor (2007)). Such generalizations are possible in our framework bringing some new insight to the behavioural contracts literature. We also extend the analysis of Woźny (2015), who considered the optimal contract of a static principal-agent model with temptations on the agents side modelled using GP method. As compared to his paper, here we show the new channels of managing agent's self control costs that are available to the principal during repeated interactions, including consumption or risk-aversion smoothing as well as savings or renegotiation. Our paper is also related to a number of other papers studying temptation / time consistency in the contractual framework. See e.g. Eliaz and Spiegel (2006), who characterize the optimal contract to screen naive agents with dynamically inconsistent preferences. Additionally, Heidhues and Koszegi (2010) analyze credit markets, when borrowers have a taste for immediate gratification. See also Della Vigna and Malmendier (2004), who characterize the optimal contract design for (partially) naive agents with  $\beta - \delta$  preferences. We also refer the reader to the interesting work of Esteban and Miyagawa (e.g., 2006), who characterize the optimal menu pricing, when consumers face temptation. Finally, we mention Gilpatric (2008) contribution, that shows, how the principal can use time-inconsistency

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<sup>1</sup>See example of Caplin and Leahy (2006) or a discussion in Balbus, Reffett, and Woźny (2015) for why it is a case.

of the naive agents' decisions to reduce the optimal cost of implementation.

The rest of the paper is organized as follows. Section 2 introduces the setting and constructs the agent's preferences. Section 3 states the main result concerning characterization of the optimal contract and comparative statics. Finally, section 4 offers few extensions of our basic model.

## 2 The model

Consider a moral hazard problem repeated over two periods. Specifically, there is a single principal and a single agent both living two periods. After accepting a contract, each period  $t$  the agent exerts a costly action  $a_t \in \{a_t^h, a_t^l\}$  that parameterize the probability distribution over each period outputs  $q_t^h, q_t^l$ . The probability of output  $i = h, l$ , when action  $a_t^j$  is chosen, is denoted by  $\pi_i(a_t^j) \in (0, 1)$  and is independent and the same for both periods. A long term contract specifies  $w = \{w_i, w_{ij}\}_{i,j=h,l}$ , i.e. wage  $w_1 = (w_h, w_l)$  in the first period dependent on observed output  $i = h, l$  and wage  $w_2 = (w_{h,h}, w_{h,l}, w_{l,h}, w_{l,l})$  in the second period dependent on the output observed in the first period (denoted by  $i = h, l$ ) and that observed in the second period (denoted by  $j = h, l$ ). In the baseline model we assume that agent does not have an access to the credit market nor can renegotiate the contract in the second period.

The principal's preferences are standard, while the agent's preferences allow for temptation. Gul and Pesendorfer (2001, 2004) show that such preferences can be represented using two utilities  $\tilde{u}, \tilde{v}$ , where  $\tilde{u}$  is a "commitment utility" function, while  $\tilde{v}$  is a "temptation utility" function. In particular, Gul and Pesendorfer show that the two period self-control preferences defined over the set of menus with a typical element  $A_t$  have a representation:

$$V_1(A_1) = \max_{a_1 \in A_1} \{\tilde{u}_1(a_1) + \tilde{v}_1(a_1) + \delta V_2(A_2)\} - \max_{a'_1 \in A_1} \tilde{v}_1(a'_1), \quad (1)$$

where  $V_2(A_2) = \max_{a_2 \in A_2} \{\tilde{u}_2(a_2) + \tilde{v}_2(a_2)\} - \max_{a'_2 \in A_2} \tilde{v}_2(a'_2)$ ,  $\delta \in (0, 1]$  is a discount factor, and  $A_2$  is determined by a pair  $(a_1, A_1)$ . The term  $\max_{a'_t \in A_t} \tilde{v}_t(a'_t) - \tilde{v}_t(a_t)$  is the cost of self control in period  $t$ . In our case, as there is no borrowing nor savings, the choice set in both period is the same, say  $A$ , and gives utility:

$$V_1(A) = \max_{a_1 \in A} \{\tilde{u}_1(a_1) + \tilde{v}_1(a_1) + \delta [\max_{a_2 \in A} \tilde{u}_2(a_2) + \tilde{v}_2(a_2) - \max_{a'_2 \in A} \tilde{v}_2(a'_2)]\} - \max_{a'_1 \in A} \tilde{v}_1(a'_1).$$

To incorporate Gul and Pesendorfer type preferences into the principal-agent model with no borrowing/savings, first note that in our setting the menu of (possibly tempting) alternatives is the set of actions available to the agent. That is, even if the agent's utility depends on actions and wages, he is tempted by items in the action set only. Next,<sup>2</sup> we model the agent's choice in two steps. First, the agent chooses whether to accept a contract or not. If contract is not

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<sup>2</sup>Following the representation of Gul and Pesendorfer (2001) preferences (see section 4).

accepted, he gets a reservation utility<sup>3</sup>  $\bar{u}$ . If, on the other hand, contract is accepted the agent gets utility  $V_1$  parameterized by the contract  $w$  and his action set is given by  $A = \{a^h, a^l\}$  each period. Then, in the second stage, after accepting a contract, the agent each period makes a choice from  $A$  and evaluates the action plan  $(a_1, a_2)$  in the maximization problem:

$$\max_{a_1 \in A} \{u_1(w_1, a_1) + v_1(w_1, a_1) + \delta [\max_{a_2 \in A} u_2(w_2, a_1, a_2) + v_2(w_2, a_1, a_2) - \max_{a'_2 \in A} v_1(w_2, a_1, a'_2)]\} - \max_{a'_1 \in A} v_1(w_1, a'_1),$$

where we explicitly parameterize utilities  $u_t, v_t$  with the obtained wages. Here observe that utility  $V_1$  is not monotone in  $A$ , if the set of actions is ordered by set inclusion. That is, costly self-control implies that preferences over action sets satisfy the so called set-betweenness axiom:  $\{a_i\} \succeq \{a_i, a_j\} \succeq \{a_j\}$ , where the order of  $i, j$  depends on how steep the contract  $w$  is.

We assume that agent's within period utility  $u_1$  is separable in reward and cost of action and given by

$$u_1(w_1, a_1) = \sum_i \pi_i(a_1) u(w_i) - c_{a_1},$$

while

$$v_1(w_1, a'_1) = \alpha \left[ \sum_i \pi_i(a'_1) u(w_i) - \bar{c}_{a'_1} \right],$$

similarly

$$u_2(w_1, a_1, a_2) = \sum_i \pi_i(a_1) \sum_j \pi_j(a_2) u(w_{ij}) - c_{a_2},$$

while

$$v_2(w_2, a_1, a'_2) = \alpha \left[ \sum_i \pi_i(a_1) \sum_j \pi_j(a'_2) u(w_{ij}) - \bar{c}_{a'_2} \right].$$

Interpreting, the cost  $c_{a^j}$  and  $\bar{c}_{a^j}$  denote the commitment and temptation costs of exerting action  $a^j \in A$  respectively, while the utility  $u$  is standard (i.e., is a differentiable, strictly increasing and strictly concave Bernoulli utility function satisfying Inada condition over random rewards).<sup>4</sup> The parameter  $\alpha \in \mathbb{R}_+$  measures the strength of temptation.

We assume that  $q^h > q^l$ , as well as that action  $a^h$  is more costly than action  $a^l$ , i.e.,  $c_{a^h} > c_{a^l}$  and  $\bar{c}_{a^h} > \bar{c}_{a^l}$ . Also, we assume that the difference in costs satisfies the following:  $\bar{c}_{a^h} - \bar{c}_{a^l} > c_{a^h} - c_{a^l}$ , which reflects the fact that it is harder to incentivize the tempted agent<sup>5</sup>. We finally assume that probabilities satisfy the following:  $1 > \pi_h(a^h) > \pi_h(a^l) > 0$ , which implies that high effort shifts probability upward in the sense of the first order stochastic dominance. Also, for the two output levels, this implies that monotone likelihood ratio property is satisfied:  $\frac{\pi_h(a^l)}{\pi_h(a^h)} < \frac{\pi_l(a^l)}{\pi_l(a^h)}$ .

To sum up, when denoting agents utility over  $w = (w_1, w_2)$  and  $a = (a_1, a_2)$  by  $U(w, a)$  we

<sup>3</sup>That can be set  $\bar{u} = V_1(\{a_i\})$ , with  $\tilde{u}_1, \tilde{u}_2, \tilde{v}_1, \tilde{v}_2$  parameterized by the fixed wage  $\bar{w}$ .

<sup>4</sup>Note, here we assume that commitment and temptation utility differ only in the costs terms, while utility from wage is the same in both. Clearly this is done for simplicity. We discuss possible generalizations in section 4.

<sup>5</sup>Note, such increasing differences assumption is critical for most results in our paper.

obtain:

$$\begin{aligned}
U(w, a) &= (1 + \alpha) \sum_i \pi_i(a_1)u(w_i) - c_{a_1} - \alpha \bar{c}_{a_1} - \alpha \max_{a'_1 \in A} \left( \sum_i \pi_i(a'_1)u(w_i) - \bar{c}_{a'_1} \right) \\
&+ \delta \pi_h(a_1) \left[ (1 + \alpha) \sum_j \pi_j(a_2)u(w_{hj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} \left( \sum_j \pi_j(a'_2)u(w_{hj}) - \bar{c}_{a'_2} \right) \right] + \\
&+ \delta(1 - \pi_h(a_1)) \left[ (1 + \alpha) \sum_j \pi_j(a_2)u(w_{lj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} \left( \sum_j \pi_j(a'_2)u(w_{lj}) - \bar{c}_{a'_2} \right) \right],
\end{aligned}$$

where  $a'_t$  denotes the maximal temptation in period  $t$ . This is a two period version of Woźny (2015) static model, with time-separable preferences and identical, independent draws of  $q_t$  each period.

Few things should be noted per this form of utility. First, observe that each period the utility consist of two parts:

- (i) the sum of commitment and temptation utility  $(1 + \alpha) \sum_i \pi_i(a_k)u(w_i) - c_{a_k} - \alpha \bar{c}_{a_k}$ , as well as
- (ii) the temptation utility evaluated (endogenously) at the optimal temptation:

$$-\alpha \max_{a'_k \in A} \left( \sum_i \pi_i(a'_k)u(w_i) - \bar{c}_{a'_k} \right).$$

Here, providing incentives to the agent has two separate effects. On the one hand, it affects the optimal choice of action determined by the first part of the utility and and the other, it influences the level of agent's utility via the second term.

Second, observe that as  $\alpha$  increases the objective gets closer to the temptation utility, and hence, in the limiting case (Strotz preferences) the choice is determined by the temptation preferences only. Third, note that  $\alpha$  increases both the cost of exerting an action as well as scales the utility  $(1 + \alpha)u$ . It does not change measures of risk aversion, however. This is important, when interpreting our results concerning the cost of providing insurance or incentives to smooth reward (in time) in the next section.

Another way of looking at the model specification is to analyse the cost of self control:

$$\alpha \left[ \sum_i \pi_i(a_k)u(w_i) - \bar{c}_{a_k} - \max_{a'_k \in A} \left( \sum_i \pi_i(a'_k)u(w_i) - \bar{c}_{a'_k} \right) \right].$$

Here, note that here are two ways available to the principal of setting the self control costs to zero. The first is to offer a flat wage, which implies that both commitment and temptation terms would be solved by the choice of a low action. The second is to provide a steep wage so that solution to both of these maximization problems is a high action. Still, note that even if commitment and temptation problem's solutions are different, the principal can influence the cost of self control by balancing incentives and insurance.

The principal's objective is given by a risk neutral utility over expected outputs and payments:

$$\begin{aligned} & \pi_h(a_1)[q^h - w_h + \delta(\pi_h(a_2)(q^h - w_{hh}) + (1 - \pi_h(a_2))(q^l - w_{hl}))] + \\ & (1 - \pi_h(a_1))[q^l - w_l + \delta(\pi_h(a_2)(q^h - w_{lh}) + (1 - \pi_h(a_2))(q^l - w_{ll}))]. \end{aligned} \quad (2)$$

### 3 The optimal incentives and self-control

#### 3.1 The optimal contract

The principal's problem is to maximize a risk neutral utility given in (2) over actions and expected payments subject to constraints. To simplify, we split the problem into two. First, we solve for the costs of implementing each action profile:  $(a^h, a^h)$ ,  $(a^h, a^l)$ ,  $(a^l, a^h)$ ,  $(a^l, a^l)$ . Second, we determine the optimal action profile.

We start with the first case. Suppose the principal wants to implement  $a_1 = a^h$  and  $a_2 = a^h$ . The ex-ante participation constraint is:

$$U(w, a^h, a^h) \geq \bar{u}.$$

Here we mention, that one way of looking at the principal's optimization problem is to observe that temptation can be seen as an outside option, that is wage dependent and hence endogenous. Clearly, the (negative) terms measuring the self control costs can be shifted to the right hand side of the participation constraint and treated as the outside option:

$$\begin{aligned} & (1 + \alpha) \sum_i \pi_i(a_1)u(w_i) - c_{a_1} - \alpha \bar{c}_{a_1} + \\ & \delta(1 + \alpha)[\pi_h(a_1)(\sum_j \pi_j(a_2)u(w_{hj}) - c_{a_2} - \alpha \bar{c}_{a_2}) + (1 - \pi_h(a_1))(\sum_j \pi_j(a_2)u(w_{lj}) - c_{a_2} - \alpha \bar{c}_{a_2})] \geq \\ & \bar{u} + \alpha \max_{a'_1 \in A} (\sum_i \pi_i(a'_1)u(w_i) - \bar{c}_{a'_1}) + \\ & \delta \alpha [\pi_h(a_1) \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{hj}) - \bar{c}_{a'_2}) + (1 - \pi_h(a_1)) \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{lj}) - \bar{c}_{a'_2})]. \end{aligned}$$

As a result, when providing incentives, the principal also (partially) controls a level of such endogenous outside option.

The second period incentive compatibility constraint gives:

$$\begin{aligned} & (1 + \alpha) \sum_j \pi_j(a^h)u(w_{ij}) - c_{a^h} - \alpha \bar{c}_{a^h} - \alpha \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{ij}) - \bar{c}_{a'_2}) \geq \\ & (1 + \alpha) \sum_j \pi_j(a^l)u(w_{ij}) - c_{a^l} - \alpha \bar{c}_{a^l} - \alpha \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{ij}) - \bar{c}_{a'_2}). \end{aligned}$$

for both  $i = l, h$ . Note the temptation term  $\alpha \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{ij}) - \bar{c}_{a'_2})$  is the same in both sides of the inequalities hence the above inequality reduces to:

$$(1 + \alpha) \sum_j \pi_j(a^h)u(w_{ij}) - c_{a^h} - \alpha \bar{c}_{a^h} \geq (1 + \alpha) \sum_j \pi_j(a^l)u(w_{ij}) - c_{a^l} - \alpha \bar{c}_{a^l}.$$

This means that temptation terms does not influence the incentive compatibility directly. However,  $\alpha$  is still present there. This inequality also implies that if the incentive compatibility constraint is binding for agent with the given level of  $\alpha$ , then agents with higher  $\alpha$ 's will not choose a high effort. Also, as  $\alpha \rightarrow \infty$  the incentive compatibility constraint mimics maximization problem of the temptation utility.

The first period incentive compatibility constraint requires:

$$\begin{aligned} & (1 + \alpha) \sum_i [\pi_i(a^h) - \pi_i(a^l)]u(w_i) + \\ & + \delta[\pi_h(a^h) - \pi_h(a^l)][(1 + \alpha) \sum_j \pi_j(a_2)u(w_{hj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{hj}) - \bar{c}_{a'_2})] + \\ & + \delta[\pi_l(a^h) - \pi_l(a^l)][(1 + \alpha) \sum_j \pi_j(a_2)u(w_{lj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} (\sum_j \pi_j(a'_2)u(w_{lj}) - \bar{c}_{a'_2})] \geq \\ & \geq c_{a^h} + \alpha \bar{c}_{a^h} - c_{a^l} - \alpha \bar{c}_{a^l}. \end{aligned}$$

Using the standard technique of replacing wages by inverse utilities  $u_i$  to calculate the cost of implementing high actions in both periods ( $C(a^h, a^h)$ ), we need to solve some strictly convex minimization problem. The presence of self control cost does not change the concavity of the problem (as our constraints are linear in the utility values  $u_i$ ). The minimization problem can, therefore, be solved using standard Kuhn-Tucker conditions. That is, by letting  $\lambda$  denote Lagrange multiplier of the participation constraint,  $\mu$  the Lagrange multiplier of (the first period) incentive compatibility constraint, while  $\tilde{\mu}_i$  the Lagrange multiplier of (the second period) incentive compatibility constraint, the necessary and sufficient condition for the optimal  $w_i$  is:

$$\frac{1}{(1 + \alpha)u'(w_i)} = \lambda[1 - \frac{\alpha}{1 + \alpha} \frac{\pi_i(a^l)}{\pi_i(a^h)}] + \mu[1 - \frac{\pi_i(a^l)}{\pi_i(a^h)}], \quad (3)$$

while the necessary and sufficient condition for the optimal  $w_{ij}$  is:

$$\frac{\pi_j(a^h)}{(1 + \alpha)u'(w_{ij})} = [\lambda + \mu(1 - \frac{\pi_i(a^l)}{\pi_i(a^h)})][\pi_j(a^h) - \frac{\alpha}{1 + \alpha} \pi_j(a^l)] + \frac{\tilde{\mu}_i}{\delta \pi_i(a^h)} [\pi_j(a^h) - \pi_j(a^l)]. \quad (4)$$

Here, observe that the principal has two ways of providing incentives. Direct one via incentive compatibility condition and the indirect (or intrinsic) one via temptation term and participation constraint. Few observations follow:

**Lemma 1** *The participation constraint is binding, i.e.  $\lambda > 0$ .*

**Proof:** Summing equations (3) for  $i = 1, 2$  we obtain:

$$\lambda = \sum_{i=1}^2 \frac{\pi_i(a^h)}{u'(w_i)}.$$

Since  $u$  is strictly increasing and  $\pi_i(a^h) > 0$  we conclude that  $\lambda > 0$ . ■

Next, before proceeding, we need to determine whether the incentive compatibility constraints (in the first and the second period) are binding or slack. In fact, with temptation and self-control, both cases are legitimate. The reason why the slack incentive compatibility constraint is possible in our model, results from the negative (temptation) terms in the utility. That is, on the one hand risk aversion makes imposing incentives costly, but on the other hand temptation term can make insurance undesirable. We do not proceed to determine the most tempting items in both periods.

**Lemma 2** *The most tempting item in both periods is  $a'_1 = a^l$  and  $a'_2 = a^l$ .*

**Proof:** We consider four cases. First, assume  $\tilde{\mu}_i = \mu = 0$ . Then equations (3) and (4) imply that  $a'_1 \neq a^h$  and  $a'_2 \neq a^h$  as otherwise wages would be flat and incentive compatibility conditions will not be satisfied.

Second, assume  $\mu > 0$  but  $\tilde{\mu}_i = 0$ . Then  $a'_2 \neq a^h$  as otherwise equation (4) implies  $w_{il} = w_{ih}$  and the second period incentive compatibility could not be satisfied. To see that  $a'_1 \neq a^h$  observe that equation (4) implies  $w_{ih} \geq w_{il}$  by assumed MLRP. Next, assume by contradiction that  $a^h = a'_1 = \arg \max_{a' \in A} \sum_i \pi_i(a')u(w_i) - \bar{c}_{a'}$ . Then,  $(1 + \alpha) \sum_i (\pi_i(a^h) - \pi_i(a^l))u(w_i) \geq (1 + \alpha)[\bar{c}_{a^h} - \bar{c}_{a^l}] > c_{a^h} - c_{a^l} + \alpha[\bar{c}_{a^h} - \bar{c}_{a^l}]$  by increasing differences in costs. But this condition, together with  $w_{ih} \geq w_{il}$ , contradicts that the first period incentive compatibility condition is satisfied with equality, as required by  $\mu > 0$ .

Third, assume  $\mu = 0$  but  $\tilde{\mu}_i > 0$ . This implies that the second period incentive compatibility constraint is satisfied with equality requiring that  $a'_2 = a^l$ , by increasing differences in costs. To see that  $a'_1 = a^l$  observe that  $\mu = 0$  implies that the expected payment for the second period is independent on first period output, and hence  $w_{hj} = w_{lj}$ . This implies that the first period incentive compatibility reduces to:

$$(1 + \alpha)[\pi_h(a^h) - \pi_h(a^l)][u(w_h) - u(w_l)] \geq c_{a^h} + \alpha\bar{c}_{a^h} - c_{a^l} - \alpha\bar{c}_{a^l},$$

for which to hold, it is necessary that  $w_h > w_l$ , and hence, by equation (3), we obtain  $a'_1 = a^l$ .

Fourth, assume  $\mu > 0$  and  $\tilde{\mu}_i > 0$ . As above, the second period incentive compatibility implies that  $a'_2 = a^l$ . To see that  $a'_1 = a^l$  observe, that otherwise  $(1 + \alpha) \sum_i (\pi_i(a^h) - \pi_i(a^l))u(w_i) > c_{a^h} - c_{a^l} + \alpha[\bar{c}_{a^h} - \bar{c}_{a^l}]$  which means that for the first period incentive compatibility to hold with

equality we must have:

$$(1 + \alpha) \sum_j \pi_j(a_2)u(w_{hj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} \left( \sum_j \pi_j(a'_2)u(w_{hj}) - \bar{c}_{a'_2} \right) < \quad (5)$$

$$(1 + \alpha) \sum_j \pi_j(a_2)u(w_{lj}) - c_{a_2} - \alpha \bar{c}_{a_2} - \alpha \max_{a'_2 \in A} \left( \sum_j \pi_j(a'_2)u(w_{lj}) - \bar{c}_{a'_2} \right), \quad (6)$$

implying  $\sum_j \pi_j(a_2)u(w_{hj}) < \sum_j \pi_j(a_2)u(w_{lj})$ . This means that the expected payment in the second period following low output in the first period, must be higher than that following the high output. But this contradicts the first order condition (4) stating that:

$$\sum_j \frac{\pi_j(a^h)}{u'(w_{hj})} = \left[ \lambda + \mu \left( 1 - \frac{\pi_h(a^l)}{\pi_h(a^h)} \right) \right] > \left[ \lambda + \mu \left( 1 - \frac{\pi_l(a^l)}{\pi_l(a^h)} \right) \right] = \sum_j \frac{\pi_j(a^h)}{u'(w_{lj})}.$$

This contradiction implies that  $a'_2 = a^l$ . ■

The previous lemma implies few points. First, the principal does not reduce the agents self-control costs to zero, i.e. he chooses an incentive scheme such that the (endogenous) tempting action is  $a^l$ , not  $a^h$ . This is done to reduce the cost of implementation. Second, we can obtain a variable pay implementing high action, even in the reduced problem without the incentive compatibility condition. Third, it means in fact that in our case, the incentive compatibility condition may be binding or not at the optimal solution. In the latter case, the moral hazard problem is mitigated by the self control one. The reason for this is that in our setting the principal trade-offs incentives, insurance or consumption smoothing but also reduction of the agents's self control cost. This provides new incentives channel for the agent (via participation constraint) and hence traditional results (like binding incentive compatibility constraints at the optimal contract) are not longer valid. Moreover, this new channel can dominate all the other (insurance, providing incentives and consumption smoothing), if  $\alpha$  is sufficiently high. Next, such intrinsic incentives (provided via self-control channel) are sufficient to mitigate the moral hazard problem, but are still bounded as the (endogenously chosen) tempting item must be lower than the one principal aims to implement. Also, when incentive compatibility is slack for an agent with a given  $\alpha$  it means that the optimal (intrinsic) incentives would suffice to induce (perhaps not optimally) a high effort also for some agents with lower self-control parameters. We next analyse the monotonicity of the optimal contract:

**Proposition 1** *We have:  $w_h > w_l$  and  $w_{ih} > w_{il}$  for any  $i$ .*

**Proof:** This is implied by equations (3) and (4), assuming MLRP and recalling results of lemma 2. ■

We now aim to verify whether the optimal contract exhibits memory and satisfies the martingale property. Rearranging equations (3) and (4) we obtain:

$$\sum_j \frac{\pi_j(a^h)}{u'(w_{ij})} = \mu(1 - \frac{\pi_i(a^l)}{\pi_i(a^h)}) + \lambda = \frac{1}{(1 + \alpha)u'(w_i)} + \lambda \frac{\alpha}{1 + \alpha} \frac{\pi_i(a_1^l)}{\pi_i(a^h)},$$

or

$$\sum_j \frac{\pi_j(a^h)}{u'(w_{ij})} - \frac{1}{u'(w_i)} = -\alpha(\lambda[1 - \frac{\pi_i(a_1^l)}{\pi_i(a^h)}] + \mu[1 - \frac{\pi_i(a^l)}{\pi_i(a^h)}]). \quad (7)$$

This equations allows to see, how does the presence of costly self control scale the divergence from the martingale property. This is summarized in the next result:

**Proposition 2** *The martingale property does not hold, unless  $\alpha = 0$ , In fact for  $\alpha > 0$ , we have:*

- (i)  $\sum_j \frac{\pi_j(a^h)}{u'(w_{ij})} - \frac{1}{u'(w_i)} > 0$ ,
- (ii)  $\sum_j \frac{\pi_j(a^h)}{u'(w_{hj})} - \frac{1}{u'(w_h)} < 0$ .

**Proof:** Lemma 2 assures that  $a_1^l = a^l$ . Then, we obtain the required inequalities from equation (7). ■

The reason, why martingale property is not satisfied, is that presence of self control problems requires the principal to provide enough incentives within a given period, to reduce the costs of self control within that period. Shifting income between the periods, on the other hand, reduces the incentives within the period and hence increase the cost of self control. Hence, in order to reduce temptations for the agent the principal provides higher within period incentives. This is perhaps counterintuitive, when confronted with the recent discussion concerning EU directives<sup>6</sup> concerning remuneration policies of risk takers in the banking industry. In fact EU recommended to use bonus caps (as a ratio of fixed salary) and bonus deferral in order to mitigate the (systemic) risk of succumbing to the short term managerial goals rather than the long term ones. Such recommendations are in line with the results of the standard repeated moral hazard models. What our analysis adds to this discussion, is that, the tempting action, which risk should be avoided is in fact the less costly one, and hence too much bonus deferral or too high bonus caps can increase the likelihood to succumbing to tempting alternatives. Finally, recall that  $\alpha$  in our formulation does not change (absolute or relative) risk aversion measures, hence our results concerning lower bonus deferral does not rely on risk aversion soothing but rather incentives smoothing. Recall, that this result is a consequence of the static form of temptation in the Gul and Pesendorfer (2004) model, and unless the cost of self control is dynamic (see e.g. Noor (2007)) the martingale property will not be satisfied.

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<sup>6</sup>See: Directive 2013/36/EU of The European Parliament and of the Council.

Similarly, we consider the other cases, i.e. when the principal wants to implement low action in both periods, or high action in one of the periods only. Observe, that the wage scheme to implement the low action in both periods is flat and the same in both periods. To implement the high action in the first period only, the second period wages are flat and the dynamics of the contract is governed by the equation:

$$\frac{1}{u'(w_i^2)} - \frac{1}{u'(w_i^1)} = -\alpha(\lambda[1 - \frac{\pi_i(a_1^l)}{\pi_i(a^h)}] + \mu[1 - \frac{\pi_i(a^l)}{\pi_i(a^h)}]).$$

If  $\alpha = 0$ , then the wage for a high (low) output today is equal to the wage tomorrow conditioned on high (low) output the first period. The higher the  $\alpha$ , the higher the difference between the first period high (low) wage and the second period wage conditioned on a high (low) output the first period. Finally to implement the high action the next period only, the first period wage is flat and the martingale property is satisfied indicating that:

$$\sum_j \frac{\pi_j(a^h)}{u'(w_{lj})} = \frac{1}{u'(w_l)} = \frac{1}{u'(w_h)} = \sum_j \frac{\pi_j(a^h)}{u'(w_{hj})}.$$

### 3.2 Comparative statics

Having characterized the four cases, we can address the question of how does temptation (parameter  $\alpha$ ) influence the cost of implementation. Observe that, on the one hand, increase in  $\alpha$  increases the costs (of self control), hence the principal needs to pay more. On the other, such increase changes the marginal utility  $((1 + \alpha)u')$  of a given reward, hence it is cheaper to provide incentives. This is apparent for example, if the incentive compatibility constraint is not binding. As a result, changes in  $\alpha$  may cause non-monotone distortions in the costs of implementation, principal's payoff and hence the actions the principal wants to implement. This also means that is it not clear, if the principal (if allowed) would choose an agent with lower or higher self control parameter  $\alpha$ . It is only unambiguous, when one separates these two channels (costs vs. marginal utility), i.e. set  $\alpha = 1$  and analyse comparative statics of the cost of implementation with respect to value  $\bar{\Delta} := \bar{c}_{a_h} - \bar{c}_{a_l}$  only). Then, the marginal utility channel is closed and the higher the  $\bar{\Delta}$  the higher the cost of implementation.

We finish this section with the following comparative statics result.

**Proposition 3** *We have the following:*

- (i) if  $\mu = 0$ , then  $\frac{u'(w_h)}{u'(w_l)}$  is decreasing in  $\alpha$ ,
- (ii) if  $\tilde{\mu}_i = 0$ , then  $\frac{u'(w_{ih})}{u'(w_{il})}$  is decreasing in  $\alpha$ ,
- (iii) if  $\tilde{\mu}_i > 0$ , then  $u(w_{ih}) - u(w_{il})$  is increasing in  $\alpha$ ,
- (iv) if  $\mu > 0$ , then  $u(w_h) - u(w_l)$  or  $u(w_{hh}) - u(w_{lh}) - (u(w_{hl}) - u(w_{ll}))$  or  $u(w_{hl}) - u(w_{ll})$  is increasing in  $\alpha$ .

**Proof:** (i) follows from equations (3). In fact we have:

$$\frac{u'(w_h)}{u'(w_l)} = \frac{1 - \frac{\alpha}{1+\alpha} \frac{\pi_l(a'_1)}{\pi_l(a^h)}}{1 - \frac{\alpha}{1+\alpha} \frac{\pi_h(a'_1)}{\pi_h(a^h)}} = \frac{1 + \alpha[1 - \frac{\pi_l(a'_1)}{\pi_l(a^h)}]}{1 + \alpha[1 - \frac{\pi_h(a'_1)}{\pi_h(a^h)}]},$$

which is decreasing in  $\alpha$  by MLRP.

Similarly we obtain (ii), for a fixed period one output  $i$  from equations (4):

$$\frac{u'(w_{ih})}{u'(w_{il})} = \frac{[\lambda + \mu(1 - \frac{\pi_i(a^l)}{\pi_i(a^h)})][1 - \frac{\alpha}{1+\alpha} \frac{\pi_l(a'_2)}{\pi_l(a^h)}]}{[\lambda + \mu(1 - \frac{\pi_i(a^l)}{\pi_i(a^h)})][1 - \frac{\alpha}{1+\alpha} \frac{\pi_h(a'_2)}{\pi_h(a^h)}]} = \frac{1 + \alpha[1 - \frac{\pi_l(a'_2)}{\pi_l(a^h)}]}{1 + \alpha[1 - \frac{\pi_h(a'_2)}{\pi_h(a^h)}]}$$

which is decreasing in  $\alpha$  by MLRP.

To see (iii) observe that the second period incentive compatibility constraint must be satisfied with equality hence:

$$\sum_j [\pi_j(a^h) - \pi_j(a^l)]u(w_{ij}) = [\pi_h(a^h) - \pi_h(a^l)][u(w_{ih}) - u(w_{il})] = \frac{1}{1 + \alpha}[c_{a^h} - c_{a^l}] + \frac{\alpha}{1 + \alpha}[\bar{c}_{a^h} - \bar{c}_{a^l}],$$

which is increasing as  $\bar{c}_{a^h} - \bar{c}_{a^l} > c_{a^h} - c_{a^l}$ .

To see (iv) observe the first period incentive compatibility constraint must be satisfied with equality hence:

$$\begin{aligned} & (\pi_h(a^h) - \pi_h(a^l))[u(w_h) - u(w_l)] + \\ & \delta(\pi_h(a^h) - \pi_h(a^l))(\pi_h(a^h) - \frac{\alpha}{1 + \alpha}\pi_h(a'_2))[u(w_{hh}) - u(w_{lh}) - (u(w_{hl}) - u(w_{ll}))] + \\ & \frac{\delta}{1 + \alpha}(\pi_h(a^h) - \pi_h(a^l))[u(w_{hl}) - u(w_{ll})] = \\ & \frac{1}{1 + \alpha}(c_{a^h} - c_{a^l}) + \frac{\alpha}{1 + \alpha}(\bar{c}_{a^h} - \bar{c}_{a^l}). \end{aligned}$$

As the right hand side is increasing in  $\alpha$  at least one of the terms on the left hand side must be increasing in  $\alpha$ . ■

## 4 Extensions

In this section we consider extensions showing how general the results obtained in section 3 are.

### 4.1 More than two actions

So far we have assumed there are only two possible actions. This has been done for simplicity. It can be shown, however, that our main results extend easily to the more actions case. To see that, consider the set of actions  $A = \{a^1, a^2, \dots, a^n\}$ , the same each period, with  $c_i, \bar{c}_i$  strictly

increasing in  $i$ . Denoting  $\pi_j(a^i)$  the probability of output  $j$  if  $a^i$  was chosen and assuming the principal wants to implement action  $a^k$  in both periods, we obtain the first order conditions:

$$\frac{1}{(1+\alpha)u'(w_i)} = \lambda \left[ 1 - \frac{\alpha}{1+\alpha} \frac{\pi_i(a_1^i)}{\pi_i(a^k)} \right] + \sum_{s \neq k} \mu_s \left[ 1 - \frac{\pi_i(a^s)}{\pi_i(a^k)} \right],$$

$$\frac{\pi_j(a^k)}{(1+\alpha)u'(w_{ij})} = \left[ \lambda + \sum_{s \neq k} \mu_s \left( 1 - \frac{\pi_i(a^s)}{\pi_i(a^k)} \right) \right] \left[ \pi_j(a^k) - \frac{\alpha}{1+\alpha} \pi_j(a_2^i) \right] + \sum_{s \neq k} \frac{\tilde{\mu}_{is}}{\delta \pi_i(a^k)} \left[ \pi_j(a^k) - \pi_j(a^s) \right].$$

Although we cannot argue that  $a' = a^1$  in both periods we can show that, if  $a' = a^m$ , then  $m < k$  in both periods. This in turn implies that we still observe two channels of providing incentives to the agent: direct one (standard) and indirect one (via reduced temptation). As a result, again the incentive compatibility constraints may be not binding. Moreover the martingale property is still not satisfied:

$$\sum_j \frac{\pi_j(a^h)}{u'(w_{ij})} - \frac{1}{u'(w_i)} = -\alpha \left( \lambda \left[ 1 - \frac{\pi_i(a_1^i)}{\pi_i(a^k)} \right] + \sum_{s \neq k} \mu_s \left[ 1 - \frac{\pi_i(a^s)}{\pi_i(a^k)} \right] \right),$$

unless  $\alpha = 0$ . Corresponding result are obtained for a continuum of actions case and two outputs each period.

## 4.2 More periods and recursive formulation

To analyse  $T$  period problem we use the recursive method. It is valid, as it was in Spear and Srivastava (1987) for example, as presence of self control does not changes the concavity of the problem. Here, for the reference, we only derive the cost of implementation of high effort in every period. So consider period  $T$  problem. By  $v$  we denote the promised utility. Let

$$V_T(v) = \max_{w_h, w_l} \sum_j \pi_j(a^h) [q^j - w_j],$$

subject to period  $T$  incentive compatibility constraint:

$$(1+\alpha) \sum_j \pi_j(a^h) u(w_j) - c_{a^h} - \alpha \bar{c}_{a^h} \geq (1+\alpha) \sum_j \pi_j(a^l) u(w_j) - c_{a^l} - \alpha \bar{c}_{a^l}$$

and the promised utility condition (with the associated Lagrange multiplier  $\lambda_T := \lambda(v)$ ):

$$(1+\alpha) \sum_j \pi_j(a^h) u(w_j) - c_{a^h} - \alpha \bar{c}_{a^h} - \alpha \max_{a_2^i \in A} \left( \sum_j \pi_j(a_2^i) u(w_j) - \bar{c}_{a_2^i} \right) \geq v.$$

Denote the argument solving this problem by  $(w_T(v, q^j))_{j=h,l}$ . With the change of variables  $(w_j = u^{-1}(u_j))$  the problem can be replaced by the strictly concave program with linear con-

straints. Therefore  $V_T$  is concave. By the envelope theorem we have

$$-V'_T(v) = \lambda_T = \sum_j \frac{\pi_j(a^h)}{u'(w_T(v, q^j))}.$$

Period  $T - 1$  problem is to

$$V_{T-1}(v) = \max_{w_h, w_l, v_h, v_l} \sum_j \pi_j(a^h)[q^j - w_j + \delta V_T(v_j)],$$

subject to period  $T - 1$  incentive compatibility constraint:

$$\sum_j [\pi_j(a^h) - \pi_j(a^l)]((1 + \alpha)u(w_j) + \delta v_j) \geq c_{a^h} + \alpha \bar{c}_{a^h} - c_{a^l} - \alpha \bar{c}_{a^l}.$$

and the promised utility condition:

$$\sum_j \pi_j(a^h)((1 + \alpha)u(w_j) + \delta v_j) - c_{a^h} - \alpha \bar{c}_{a^h} - \alpha \max_{a' \in A} (\sum_j \pi_j(a')u(w_j) - \bar{c}_{a'}) \geq v.$$

Similarly we obtain the optimal solution  $(w_{T-1}(v, q^j), v_T(v, q^j))_{j=h,l}$  and that

$$-V'_{T-1}(v) = \lambda_{T-1} = \sum_j \frac{\pi_j(a^h)}{u'(w_{T-1}(v, q^j))}.$$

Proceeding this way we obtain  $V_1(\bar{u})$  and we track  $(w_t(v_t, q^j), v_{t+1}(v_t, q^j))_{j=h,l}^T$  with  $v_{T+1}(\cdot) = 0$ . Now the first order conditions for  $w_t$  are:

$$\frac{1}{(1 + \alpha)u'(w_t(v, q^j))} = \lambda_t [1 - \frac{\alpha}{1 + \alpha} \frac{\pi_j(a^l)}{\pi_j(a^h)}] + \mu_t [1 - \frac{\pi_j(a^l)}{\pi_j(a^h)}]$$

while the first order conditions for  $v_j$  are:

$$\delta \pi_j(a^h) V'_{t+1}(v) = \delta \mu [\pi_j(a^h) - \pi_j(a^l)] + \delta \lambda_t \pi_j(a^h),$$

which gives:

$$\sum_j \frac{\pi_j(a^h)}{u'(w_t(v, q^j))} = -V'_{t+1}(v) = \mu_t (1 - \frac{\pi_j(a^l)}{\pi_j(a^h)}) + \lambda_t = \frac{1}{(1 + \alpha)u'(w_t(v, q^j))} + \lambda \frac{\alpha}{1 + \alpha} \frac{\pi_j(a^l)}{\pi_j(a^h)},$$

or

$$\sum_j \frac{\pi_j(a^h)}{u'(w_t(v, q^j))} - \frac{1}{u'(w_{t-1}(v, q^i))} = -\alpha (\lambda_{t-1} [1 - \frac{\pi_i(a^l_{t-1})}{\pi_i(a^h)}] + \mu_{t-1} [1 - \frac{\pi_i(a^l)}{\pi_i(a^h)}]),$$

which is the counterpart of the two period model analysed so far. Hence, the qualitative results we have obtain for two periods are easily extendable for more  $T \geq 2$  periods, which is an

advantage of our approach, versus one modelling temptations using the time-consistency model ( $\beta - \delta$  or alike).

### 4.3 Limited commitment

Suppose now, we consider a limited commitment on the agent side. To do that, we add an additional feasibility constraint in the second period. To see how the limited commitment influence the optimal contract, observe that increase in  $\alpha$  reduces the willingness to smooth incentives. As a result, the higher the  $\alpha$  the lower the willingness to break the contract the next period. It is clear for the Strotz case ( $\alpha = \infty$ ). But in fact, if  $\alpha$  is high enough, so that the incentive compatibility constraint in the first period is satisfied but not binding, then  $v_h = v_l$ . As a result, the optimal contract, with and without commitment on the agent side, is the same. In such a case, the optimal dynamic contract is hence spot implementable (see Chiappori, Macho, Rey, and Salanie (1994)). This contradicts with a time-inconsistent (e.g.  $\beta - \delta$  model) representation of agent's preferences, where some form of commitment can be beneficial for the planner incarnation of the agent.

### 4.4 Access to the credit market

Finally, we discuss how does temptation interacts with (agent's) access to the credit market. For this reason, we modify the model to allow the agent to save, after choosing an action  $a$  and obtaining wage  $w$ . Recalling construction of the agent's utility in (1), suppose first that possibility of shifting income between periods is not tempting. Then, similarly to the  $\alpha = 0$  case, we can show that the agent would like to save after  $w_h$ . In fact, for any monotone second period wage, and principal aiming to implement  $(a^h, a^h)$  we have:

$$\frac{1}{u'(w_h)} > \sum_j \frac{\pi_j(a^h)}{u'(w_{hj})} > \sum_j \pi_j(a^h) u'(hj) > [(1 + \alpha)\pi_h(a_h) - \alpha\pi_h(a_2^t)]u(w_{hh}) + [(1 + \alpha)\pi_l(a_h) - \alpha\pi_l(a_2^t)]u(w_{hl}).$$

Observe that period 2 temptation is still  $a_l$  (after transfer), as the richer the agent gets the lower the action he chooses. Moreover, such transfer (saving) will not necessarily alter the second period action. This is due to the fact, that the incentive compatibility constraint in the second period was not necessarily binding before.

Next, suppose we go back to the full specification of the agent's utility in (1), and additionally add temptation to consume the whole income each period (say no borrowing is allowed). Then, in fact agent may want to save less or not at all, as temptations increase the marginal utility  $(1 + \alpha)u'$ . For this reason, the direction in which  $\alpha$  influences incentives to save is ambiguous.

## 4.5 General temptation utility

Finally, our results can be extended to more general preferences case. In fact, our model so far has general commitment preferences but specific temptation preferences. We have chosen this formulation to parameterize the cost of self control. More general, however, the temptation utility  $v$  may be different from commitment one, namely  $u$ . To extend our qualitative results obtained in section 3 to allow for more general temptation utility we need to assume that  $v$  is convex, but such that  $u + v$  is strictly concave. Moreover, apart from assuming increasing differences in costs (namely:  $\bar{c}_{a^h} - \bar{c}_{a^l} > c_{a^h} - c_{a^l}$ ) we also require increasing differences in preferences:  $u(w_2) - u(w_1) \geq v(w_2) - v(w_1)$  for any  $w_2 \geq w_1$ . These two assumptions are sufficient that the principal problem (and the value function in multiperiod case) remains concave and that it is more costly to provide incentives to the tempted agent.

## 5 Conclusions

In this paper we have discussed costly self control implications on the shape of optimal dynamic contract in the repeated principal agent model with moral hazard. We have characterized the optimal contract and identified the new channel of principal and agents interactions that can be used to provide incentives to the agent, this being the reduction of its self control costs. In fact, the principal computes and uses agent's optimal temptation but never finds it optimal to reduce the agent's self-control cost to zero. One way of looking at the principal's optimization problem in our case is to observe that temptation can be seen as an outside option, that is wage dependent and hence endogenous<sup>7</sup>. As a result, when providing incentives the principal, also (partially) controls a level of such endogenous outside option.

Presence of this new channel challenges typical results obtained in models with no-temptation on the agent's side. Traditional results (like binding incentive compatibility constraints at the optimal contract) for models without temptations are not longer valid<sup>8</sup>. This implies, for example, that the intrinsic motivation (resulting from costly self-control) can substitute for standard (external) incentives, and hence the moral hazard problem can be mitigated (for sufficiently high temptation). Moreover, the optimal contract calls for a lower bonus deference (or consumption smoothing) between the periods. This new channel can dominate all the other (insurance, providing incentives and consumption smoothing), if parameter measuring the strength of temptations, i.e.  $\alpha$  is sufficiently high. Finally, such intrinsic incentives (provided via self-control channel) are sufficient to mitigate the moral hazard problem, but are still bounded even for the limiting (Strotz) case ( $\alpha \rightarrow \infty$ ).

From the technical perspective, our approach of modelling temptations using GP framework has advantages over using time-inconsistent preferences, that are especially visible in the multi-

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<sup>7</sup>For a different class of models, where outside option is endogenous, see e.g. Atkeson (1991), Grochulski and Zhang (2016) or earlier literature on career concerns.

<sup>8</sup>This is also true in Engmaier and Leider (2012) paper on the optimal contract with reciprocal agents.

period case. Moreover, self-control also reduces agent's willingness to break or renegotiate the contract after the first period, and make the optimal contract spot implementable (again for sufficiently high temptation). Impact of self-control on the cost of implementation as well as willingness to save/borrow is ambiguous.

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