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Abstract

When some steps of a complex, multi-step task are automated, the demand for human work in the remaining complementary sub-tasks goes up. In contrast, when the task is fully automated, the demand for human work declines. Partial automatability of complex tasks leads to a bottleneck of development (where further growth is constrained by the scarcity of essential human work) which is removed once the tasks become fully automatable. Theoretical analysis using a two-level nested CES production function specification demonstrates that the shift from partial to full automation generates a non-convexity: humans and machines switch from complementary to substitutable, and the share of output accruing to human workers switches from an upward to a downward trend. This process has implications for inequality, the risk of technological unemployment and the likelihood of a secular stagnation.

Keywords: automation, complex task, complementarity, factor share, nested CES.

JEL codes: J23, L11, O30, O40.

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1 Introduction

Thinking about the long-run impact of automation on the economy, the threat of technological unemployment or the likelihood of an upcoming secular stagnation, it is important to account also for mechanisms that may not have been important in the past but are likely to intensify in the future. In this paper I discuss one such mechanism: *a shift from partial to full automation of complex tasks*. The key theoretical insight of this paper is as follows: if a task is complex – that is, requires completion of at least two complementary sub-tasks – then it makes a crucial difference if only some or all of the sub-tasks are automatable. Automating some but not all sub-tasks increases the relative value of (and returns to) sub-tasks that cannot be automated. Automating all steps undoes this effect. Partial automatability makes people and programmable machines complementary, whereas full automatability makes them substitutable. For this reason, growing wages and stable employment are safe only when full automation is technologically infeasible. For the very same reason, though, achieving full automatability of complex tasks can generate a permanent boost to output growth.

Think for example of the photography industry, represented by companies of two technological eras: Kodak and Instagram. "Kodak was founded in 1880, and at its peak employed nearly 145,300 people, with many more indirectly employed via suppliers and retailers. Kodak's founding family, the Eastmans, became wealthy, while providing skilled jobs for several generations of middle-class Americans. Instagram was founded in 2010 by a team of fifteen people. In 2012 it was sold to Facebook for over one billion dollars. Facebook, worth far more than Kodak ever was, employs fewer than 5,000 people. At least ten of them have a net worth ten times that of George Eastman."[1] Gradual technological improvements and partial automation in the photography industry have been benefiting companies like Kodak for decades, increasing their employment and the overall wage bill. By contrast, full automation exemplified by Instragram reversed this trend. Once the entire multi-step task of providing the service – in this case sharing a certain visual experience with oneself and others – could be provided to the consumer without any human input, employment and the wage bill in the protography industry plummeted. What rose instead was the returns to the automation technology (computer software) and, in effect, profits and shareholder value of companies like Instragram.

A similar case to consider is retail sales of books. "When Amazon.com sold its first book 20 years ago, Borders Books & Music had a thriving retail empire generating about $1.6 billion a year in sales. Today, Borders is nothing but a memory, ushered to the grave by an e-commerce revolution led by Amazon."[2] And again,

[1] https://www.newstatesman.com/politics/2014/01/kodak-vs-instagram-why-its-only-going-get-harder-make-good-living [access: 13.12.2019]

[2] https://www.sfchronicle.com/business/article/How-Amazon-factor-killed-retailers-like-

it is not that Borders did not automate at all: they adopted electronic inventory management, invoicing systems, put up an online catalog, etc. But in their case automation was only partial, and they did not make the final step which Amazon did: offering the entire service – putting a certain book in a certain person's hand – without any human input. Just like in the photography industry, the real game-changer here was the shift from partial to full automation. Amazon's new technology was *disruptive*.

The key reason to believe that in the coming decades we will see many more industries disrupted by a shift from partial to full automation is that growth in the digital sphere, responsible for all progress in automation, is now an order of magnitude faster than growth in the global capital stock and GDP: data volume, processing power and bandwidth double every 2–3 years, whereas global GDP doubles every 20–30 years. In particular, since the 1980s "general-purpose computing capacity grew at an annual rate of 58%. The world's capacity for bidirectional telecommunication grew at 28% per year, closely followed by the increase in globally stored information (23%)" (Hilbert and López, 2011). The costs of a standard computation have been declining by 53% per year on average since 1940 (Nordhaus, 2017). The processing, storage, and communication of information has decoupled from the cognitive capacities of the human brain: "less than one percent of information was in digital format in the mid-1980s, growing to more than 99% today" (Gillings, Hilbert, and Kemp, 2016). Preliminary evidence also suggests that since the 1980s the efficiency of computer algorithms has been improving at a pace that is of the same order of magnitude as accumulation of digital hardware (Grace, 2013). Corroborating this finding, in the recent decade we have witnessed a surge in AI breakthroughs based on the methodology of *deep neural networks* (Tegmark, 2017), from autonomous vehicles and simultaneous language interpretation to self-taught superhuman performance at chess and Go (Silver, Hubert, Schrittwieser, et al., 2018).

In the current paper I formalize the consequences of a shift from partial to full automation with a simple model of tasks that consist of two sub-tasks. I assume that sub-tasks within a task are complementary, whereas within each sub-task people and machines are substitutable. I use this model to compare the "partial automation" scenario where some of the sub-tasks can be performed only by a human against a "full automation" scenario where all sub-tasks can be performed both by a human and a pre-programmed machine. I assume perfect competition at factor markets and perfect mobility of factors across the sub-tasks, so that their remuneration in both sub-tasks is equalized. In each of the considered scenarios I compute the equilibrium allocation of factors across sub-tasks, factor shares and wage rates. I then analyze how these numbers change with technological progress and the accumulation of

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programmable machines able to perform automatable tasks. (My results can be easily generalized to tasks consisting of an arbitrary number of sub-tasks, some of which are automatable and some are not.)

I find that when at least one sub-task is not automatable, progress in automation makes the scarce human input increasingly valuable, thereby increasing wages and the labor share of output towards unity. Then, as the extent of automation becomes sufficiently high, human work becomes the bottleneck of further economic growth. When all tasks are automatable, in contrast, progress in automation makes the scarce human input increasingly *less* valuable, decreasing the labor share of output towards zero. As automation technology becomes sufficiently advanced, human work becomes unimportant for production. Economic growth continues unabated, but its fruit are increasingly captured by (owners of) pre-programmed machines and their software, not the human workers. The shift from partial to full automatability of tasks creates a non-convexity in economic development, where human and machine inputs switch from complementary to substitutable (in the sense of the aggregate elasticity of substitution, cf. Miyagiwa and Papageorgiou (2007); Xue and Yip (2013)). While boosting growth, it also generates a secular upward trend in inequality by gradually redirecting income from a wide population of workers to a relatively narrow group of owners of programmable machines and their software.

The results obtained in the current study are helpful in answering the important question whether automation will bring technological unemployment. Will a robot take your job? Will humans go the way of horses? Does technological progress destroy fewer or more jobs than it creates? (Brynjolfsson and McAfee, 2014; Frey and Osborne, 2017; Autor and Salomons, 2018) On past evidence, the overall balance has been positive thus far: even if routine jobs were succumbing to automation (Autor and Dorn, 2013), these falls has been compensated – and in aggregate value terms, more than compensated – by the rise of high-skill, non-routine cognitive tasks and occupations ("frontier jobs", Autor, 2019) as well as the auxiliary low-skilled ones ("wealth work" and "last mile jobs"). However, this conclusion is not guaranteed to persist: as more and more sectors become fully automated, even these jobs may eventually disappear.

The current paper also informs the debate on the future of global economic growth – whether we should expect secular stagnation (Jones, 2002; Gordon, 2016), further exponential growth, or a technological singularity (Kurzweil, 2005). My key point is that secular stagnation requires setting a firm limit to automatability: for such a scenario to materialize there must exist an essential task in the economy, complementary to all others, which cannot be fully automated. Otherwise, production will get increasingly automated and aggregate output will gradually decouple from human work, becoming instead proportional to the work done by pre-programmed machines.

The current study is related more broadly to studies focusing on automation and its impacts on productivity, employment, wages and factor shares (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2018; Andrews, Criscuolo, and Gal, 2016; Arntz, Gregory, and Zierahn, 2016; Frey and Osborne, 2017; Barkai, 2017; Autor, Dorn, Katz, Patterson, and Van Reenen, 2017; Jones and Kim, 2018; Hemous and Olsen, 2018). It also touches the nascent literature on macroeconomic implications of development of "digital labor", AI and autonomous robots (Yudkowsky, 2013; Graetz and Michaels, 2018; Sachs, Benzell, and LaGarda, 2015; Benzell, Kotlikoff, LaGarda, and Sachs, 2015; DeCanio, 2016; Acemoglu and Restrepo, 2018; Aghion, Jones, and Jones, 2019; Berg, Buffie, and Zanna, 2018; Benzell and Brynjolfsson, 2019). In particular Benzell and Brynjolfsson (2019) consider a model where automation replaces capital and labor but is complementary to a scarce factor "genius". The predictions of this model are very similar to the "partial automation" scenario, with "genius" acting as the human work necessary for carrying out the non-automatable sub-task.

Last but not least, the current paper can also be viewed in conjunction with my other one (Growiec, 2019) in which I formalize the distinction between mechanization and automation with the hardware–software model. What I refer to as "human and machine work" in the current paper is equivalent to "human cognitive work and pre-programmed software" within the *software* factor discussed there. Keeping this in mind, it is straightforward to observe that mechanization initiated in the Industrial Revolution had vastly different implications for factor shares than automation which began with the Digital Revolution: the former featured replacement of humans with machines in the hardware factor (brawn) whereas the latter pertains to the software factor (brains). Mechanization raised demand for human cognitive work, automation replaces it. Demand for human cognitive work can go up only to the extent it is complementary to the automated tasks, i.e., only as long as automation is partial.

2 Model of Partial and Full Automation

2.1 Setup

Consider a task T consisting of two complementary sub-tasks T_1 and T_2,

$$T = T_0 \left(\pi_0 \left(\frac{T_1}{T_{01}} \right)^{\varepsilon} + (1-\pi_0) \left(\frac{T_2}{T_{02}} \right)^{\varepsilon} \right)^{\frac{1}{\varepsilon}}. \tag{1}$$

Output of task T is thus modeled with a normalized CES function with constant returns to scale (Klump and de La Grandville, 2000; Klump, McAdam, and Willman, 2012). The parameter $\varepsilon < 0$ signifies gross complementarity of the two sub-tasks, linking to the elasticity of substitution via $\sigma = \frac{1}{1-\varepsilon} \in (0, 1)$. The parameter

$\pi_0 \in (0,1)$ is the share of sub-task 1 in the total output of task T at the point of normalization. Variables with subscript 0 are (positive) normalization constants.

I would like to compare two scenarios: (i) *partial automation*, where sub-task 2 is not automatable and can be performed by humans only, and (ii) *full automation*, where both tasks can be performed both by humans L and programmable machines K. Taking again the normalized CES form, output of each of the subtasks can be written as:

$$T_i = T_{0i}\left(\pi_{0i}\left(\psi_i\frac{K}{K_0}\right)^{\theta} + (1-\pi_{0i})\left(n_i\frac{L}{L_0}\right)^{\theta}\right)^{\frac{1}{\theta}}, \qquad i = 1,2, \tag{2}$$

where $\theta \in (0,1]$ signifies gross substitutability of humans and machines in each of the two sub-tasks. In the polar case $\theta = 1$ both inputs are perfectly substitutable in production. $K > 0$ is the total supply of machines in the economy whereas $L > 0$ is total employment of people. The parameter ψ_i captures the productivity-adjusted share of machines employed in performing sub-task i, with $\psi_1 + \psi_2 = \psi$ fixed. Analogously, n_i captures the productivity-adjusted share of people employed in performing sub-task i, with $n_1 + n_2 = n$ fixed.

The number ψ represents the unit productivity of programmable machines K, and thus increases in ψ represent progress in efficiency of machine work, stemming e.g. from improved machine architecture or improved algorithms. The overall *progress in automation* is captured by growth in the product ψK, which has both the intensive margin (growth in ψ), and the extensive margin (growth in K, e.g. increases in raw computing power).

In turn, n represents the number of productivity-adjusted hours worked per worker. Increases in n may thus represent either increases in average hours worked or in the average unit productivity of an hour worked.

In this notation, the partial automation case is a constrained variant of the full automation case, obtained by imposing $\psi_2 = 0$ and thus $\psi_1 = \psi$. Moreover, it is easily observed that the current setup can be understood as encompassing any finite number of sub-tasks: "sub-task 1" is a catch-all term covering all sub-tasks which are automatable.

Under perfect competition and constant returns to scale, the shares of sub-tasks 1 and 2 in output sum up to unity. The shares of humans and machines in each of the sub-tasks sum up to unity, too. Under the normalized CES specification the shares are computed as follows:

$$\pi = \pi_0\left(\frac{T_1}{T_{01}}\frac{T_0}{T}\right)^{\varepsilon}, \qquad \text{share of sub-task 1,} \tag{3}$$

$$\pi_1 = \pi_{01}\left(\frac{\psi_1 K}{K_0}\frac{T_{01}}{T_1}\right)^{\theta}, \qquad \text{machines share in sub-task 1,} \tag{4}$$

$$\pi_2 = \pi_{02}\left(\frac{\psi_2 K}{K_0}\frac{T_{02}}{T_2}\right)^{\theta}, \qquad \text{machines share in sub-task 2.} \tag{5}$$

The overall machines share of output is $\pi_K = \pi\pi_1 + (1-\pi)\pi_2$, and the human labor share is $\pi_L = 1 - \pi_K = \pi(1-\pi_1) + (1-\pi)(1-\pi_2)$.

Using normalized intensive units, $k = \frac{K}{K_0}\frac{L_0}{L}, t_i = \frac{T_i}{T_{0i}}\frac{L_0}{L}, t = \frac{T}{T_0}\frac{L_0}{L}$, we obtain: $\pi = \pi_0(t_1/t)^\varepsilon$, and for $i = 1, 2$, $\pi_i = \pi_{0i}(\psi_i k/t_i)^\theta$.

In any interior solution, equalization of wages across sub-tasks 1 and 2 ($w_1 = w_2$) yields:

$$w_1 = \frac{\partial T}{\partial(n_1 L)} = \frac{\partial T}{\partial(n_2 L)} = w_2 \qquad \Longleftrightarrow \qquad \frac{n_1}{n_2} = \frac{\pi}{1-\pi}\frac{1-\pi_1}{1-\pi_2}. \tag{6}$$

Furthermore, if both sub-tasks are automatable, equalization of rental rates of machines across sub-tasks 1 and 2 ($r_1 = r_2$) yields:

$$r_1 = \frac{\partial T}{\partial(\psi_1 K)} = \frac{\partial T}{\partial(\psi_2 K)} = r_2 \qquad \Longleftrightarrow \qquad \frac{\psi_1}{\psi_2} = \frac{\pi}{1-\pi}\frac{\pi_1}{\pi_2}. \tag{7}$$

If sub-task 2 is not automatable, equation (7) ceases to hold and all machines are allocated to sub-task 1 where they are remunerated according to their marginal product.

Dealing with the long run implications of both scenarios, I am going to be particularly concerned with the impact of *progress in automation*, which I will understand as an increase in ψk keeping n constant. By the *extent of automation* I will consequently mean the ratio $\psi k/n$.

2.2 Results under Perfect Substitutability

I shall first assume perfect substitutability of people and machines within each sub-task, $\theta = 1$. This special case is particularly transparent insofar as it implies that in equilibrium, output at the level of the whole task follows a normalized CES function, too.

2.2.1 Partial Automation: Sub-Task 2 Not Automatable

When $\theta = 1$, from wage equalization (6) we get:

$$\frac{t_1}{t_2} = \left(\frac{\pi_0}{1-\pi_0}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\varepsilon}}. \tag{8}$$

Hence, the interior equilibrium requires that output from sub-tasks 1–2 must come in a fixed proportion. In the following analysis I will denote this ratio as $\xi > 0$.

Equilibrium allocation. Assuming $\psi_2 = 0$ and thus $\psi_1 = \psi$, and therefore $t_1 = \pi_{01}(\psi k) + (1-\pi_{01})n_1$ and $t_2 = (1-\pi_{02})n_2$, I obtain that employment in sub-task 1 equals:

$$n_1 = \begin{cases} \frac{\xi(1-\pi_{02})n - \pi_{01}(\psi k)}{\xi(1-\pi_{02}) + (1-\pi_{01})}, & \text{if } \frac{\psi k}{n} \le \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ 0, & \text{if } \frac{\psi k}{n} > \frac{\xi(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{9}$$

If the extent of automation $\psi k/n$ is low (below a certain exogenous threshold), human work is used in both sub-tasks. The economy is then in an interior equilibrium. If the extent of automation is high, though, the economy finds itself in a corner equilibrium and human work is used only in sub-task 2 which is not automatable.

Factor shares and substitutability. If $\psi_2 = 0$ and thus machines are not employed in sub-task 2, the relative factor share in the economy, i.e., the ratio of the machines share π_K to the human labor share π_L equals:

$$\Pi = \frac{\pi_K}{\pi_L} = \frac{\pi\pi_1}{\pi(1-\pi_1)+(1-\pi)} = \frac{\pi_0\pi_{01}t_1^{\varepsilon-1}(\psi k)}{\pi_0(1-\pi_{01})t_1^{\varepsilon-1}n_1+(1-\pi_0)t_2^{\varepsilon}}. \tag{10}$$

Hence,

$$\Pi = \begin{cases} \left(\frac{\pi_{01}}{1-\pi_{01}}\right)\frac{\psi k}{n}, & \text{if } \frac{\psi k}{n} \leq \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ \left(\frac{\pi_{01}}{1-\pi_{01}}\right)\left(\frac{\pi_{01}\psi k}{(1-\pi_{02})n}\right)^{\varepsilon}, & \text{if } \frac{\psi k}{n} > \frac{\xi(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{11}$$

Equation (11) has very important implications. It elucidates that as long as the extent of automation is low, human and machine work are perfectly substitutable at the level of the whole task because they are substitutable at the level of each sub-tasks and there is a degree of freedom to keep the ratio of both tasks fixed in equilibrium. If the extent of automation is high, though, this degree of freedom is no longer present. When all human work is allocated to the non-automatable sub-task 2, it becomes complementary to machines because the human-operated sub-task 2 is complementary to the machine-operated sub-task 1.

It is also instructive to compute the equilibrium wage rate, which is equal to:

$$w = w_2 = (1-\pi)(1-\pi_2)\frac{T}{n_2 L} = (1-\pi_0)(1-\pi_{02})\left(\frac{t_2}{t}\right)^{\varepsilon-1}\frac{T_0}{L_0}, \tag{12}$$

and thus is proportional to the contribution of the non-automatable sub-task 2 to overall output, t_2/t.

Inserting the equilibrium allocation of human work into final task output we obtain that in equilibrium the normalized CES form with human and machine work is reproduced at the level of the whole task, with an infinite elasticity of substitution if $\psi k/n$ is low, and a low elasticity of substitution $\frac{1}{1-\varepsilon} < 1$ if $\psi k/n$ is high:

$$t = \begin{cases} (\pi_0 + (1-\pi_0)\xi^{-\varepsilon})^{\frac{1}{\varepsilon}}(\pi_{01}(\psi k) + (1-\pi_{01})n_1), & \text{if } \frac{\psi k}{n} \leq \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ (\pi_0(\pi_{01}\psi k)^{\varepsilon} + (1-\pi_0)((1-\pi_{02})n)^{\varepsilon})^{\frac{1}{\varepsilon}}, & \text{if } \frac{\psi k}{n} > \frac{\xi(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{13}$$

Impact of automation. As ψk goes up (reflecting technological progress and the accumulation of programmable machines able to perform sub-task 1), eventually it must cross the threshold $\xi(1-\pi_{02})n/\pi_{01}$. From that moment on we arrive at the corner solution where all human work is allocated to the non-automatable sub-task 2, making human and machine work complementary (with a low elasticity of substitution $\frac{1}{1-\varepsilon}$). The human labor share of output grows, eventually to unity as $\psi k \to \infty$. Wages grow in negative sync with the declining contribution of sub-task 2 to overall output (t_2/t), mirroring the increasing scarcity of human work, but eventually converge to a firm upper bound.

Long-run steady state. In the long run steady state (in which $\psi k \to \infty$), all human work is allocated to the non-automatable sub-task 2 and the human labor share of output π_L is one. Output approaches the upper limit:

$$t_{\max} = (1-\pi_0)^{\frac{1}{\varepsilon}}(1-\pi_{02})n. \tag{14}$$

In consequence, wages approach their respective upper limit:

$$w_{\max} = (1-\pi_0)^{\frac{1}{\varepsilon}}(1-\pi_{02})\frac{T_0}{L_0}. \tag{15}$$

In the long run, human work is a bottleneck of development. Total output is bounded above and further growth is impossible. The only way to circumvent this "underdevelopment trap" is to make all sub-tasks automatable, rendering the human input no longer essential for production.

2.2.2 Full Automation: Both Sub-Tasks Automatable

When both sub-tasks are automatable, both people and machines can be freely allocated to either of the two sub-tasks. In any interior equilibrium, wages and rental rates of machines must be equalized across the sub-tasks (equations (6)–(7)), implying that

$$\frac{n_1}{n_2} = \frac{\psi_1}{\psi_2}\left(\frac{\pi_2}{\pi_1}\frac{1-\pi_1}{1-\pi_2}\right). \tag{16}$$

However, further inspection reveals that with perfect substitutability of people and machines within sub-tasks, equation (16) is either trivially satisfied if $\pi_{01} = \pi_{02}$ or otherwise leads to a contradiction. It is more constructive to concentrate on the typical case $\pi_{01} \neq \pi_{02}$. In analyzing this case, without loss of generality we may assume $\pi_{01} > \pi_{02}$, so that sub-task 1 is relatively more machine-intensive than sub-task 2. In this case there is no interior equilibrium. Instead, three types of corner equilibria are possible: (i) with machines in both sub-tasks, humans only in sub-task 2, (ii) with machines only in sub-task 1 and humans only in sub-task 2, and (iii) with machines only in sub-task 1 and humans in both sub-tasks. The choice of equilibrium will depend critically on the extent of automation, $\psi k/n$.

Equilibrium allocation. The following allocation of human work across sub-tasks is derived:

$$n_1 = \begin{cases} \frac{\xi(1-\pi_{02})n-\pi_{01}(\psi k)}{\xi(1-\pi_{02})+(1-\pi_{01})}, & \text{if } \frac{\psi k}{n} \leq \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ 0, & \text{if } \frac{\psi k}{n} > \frac{\xi(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{17}$$

In turn, the allocation of machines is:

$$\psi_1 k = \begin{cases} \psi k, & \text{if } \frac{\psi k}{n} \leq \frac{\zeta(1-\pi_{02})}{\pi_{01}}, \\ \frac{\zeta((1-\pi_{02})n+\pi_{02}(\psi k))}{\zeta\pi_{02}+\pi_{01}}, & \text{if } \frac{\psi k}{n} > \frac{\zeta(1-\pi_{02})}{\pi_{01}}, \end{cases} \tag{18}$$

where $\xi = \left(\frac{\pi_0}{1-\pi_0}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\varepsilon}} < \left(\frac{\pi_0}{1-\pi_0}\frac{\pi_{01}}{\pi_{02}}\right)^{\frac{1}{1-\varepsilon}} = \zeta$ because $\pi_{01} > \pi_{02}$.

If the extent of automation is low ($\psi k/n$ below the lower threshold), human work is used in both sub-tasks and machines are used only in sub-task 1. For intermediate values of $\psi k/n$ there is perfect specialization, so that sub-task 1 employs only machines, and sub-task 2 employs only people. Finally, if the extent of automation is high ($\psi k/n$ above the higher threshold), human work is used only in sub-task 2 while machines are employed in both sub-tasks.

If $\pi_{01} = \pi_{02}$ then $\xi = \zeta$ and the intermediate case disappears.

Factor shares and substitutability. When both tasks are automatable, the relative factor share Π equals:

$$\Pi = \frac{\pi_K}{\pi_L} = \frac{\pi\pi_1 + (1-\pi)\pi_2}{\pi(1-\pi_1) + (1-\pi)(1-\pi_2)} \tag{19}$$

$$= \frac{\pi_0\pi_{01}t_1^{\varepsilon-1}(\psi_1 k) + (1-\pi_0)\pi_{02}t_2^{\varepsilon-1}(\psi_2 k)}{\pi_0(1-\pi_{01})t_1^{\varepsilon-1}n_1 + (1-\pi_0)(1-\pi_{02})t_2^{\varepsilon-1}n_2}. \tag{20}$$

Hence,

$$\Pi = \begin{cases} \left(\frac{\pi_{01}}{1-\pi_{01}}\right)\frac{\psi k}{n}, & \text{if } \frac{\psi k}{n} \leq \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ \left(\frac{\pi_{01}}{1-\pi_{01}}\right)\left(\frac{\pi_{01}\psi k}{(1-\pi_{02})n}\right)^{\varepsilon}, & \text{if } \frac{\psi k}{n} \in \left(\frac{\xi(1-\pi_{02})}{\pi_{01}}, \frac{\zeta(1-\pi_{02})}{\pi_{01}}\right), \\ \left(\frac{\pi_{02}}{1-\pi_{02}}\right)\frac{\psi k}{n}, & \text{if } \frac{\psi k}{n} \geq \frac{\zeta(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{21}$$

Equation (21) signifies that when both tasks are automatable, human and machine work are perfectly substitutable at the level of the whole task both when the extent of automation is low and when it is high. Complementarity occurs only in the intermediate case of full specialization, where all human work is allocated to sub-task 2 and all machines operate in sub-task 1. Unlike the *partial automation* scenario, this result is however reversed once the extent of automation crosses the upper threshold $\zeta(1-\pi_{02})/\pi_{01}$. From that moment onwards, a new degree of freedom is opened – machines can now be freely allocated across both tasks, and in equilibrium they are allocated such that the contribution of each sub-task is fixed (t_1/t and t_2/t are constant).

The equilibrium wage rate still follows equation (12) and thus is proportional to the contribution of sub-task 2 to overall output, t_2/t. Following the results above, however, it is now constant regardless of factor endowments:

$$w = (1-\pi_0)(1-\pi_{02})\left(\pi_0\zeta^{\varepsilon} + (1-\pi_0)\right)^{\frac{1-\varepsilon}{\varepsilon}}\frac{T_0}{L_0}. \tag{22}$$

Again, inserting the equilibrium allocation of people and machines into the final task output we obtain that in equilibrium the normalized CES form with human and machine work is reproduced at the level of the whole task, with an infinite elasticity of substitution if ψk is low or high, and a low elasticity of substitution $\frac{1}{1-\varepsilon} < 1$ if ψk takes intermediate values:

$$t = \begin{cases} (\pi_0 + (1-\pi_0)\xi^{-\varepsilon})^{\frac{1}{\varepsilon}}(\pi_{01}(\psi k) + (1-\pi_{01})n_1), & \text{if } \frac{\psi k}{n} \leq \frac{\xi(1-\pi_{02})}{\pi_{01}}, \\ (\pi_0(\pi_{01}\psi k)^{\varepsilon} + (1-\pi_0)((1-\pi_{02})n)^{\varepsilon})^{\frac{1}{\varepsilon}}, & \text{if } \frac{\psi k}{n} \in \left(\frac{\xi(1-\pi_{02})}{\pi_{01}}, \frac{\zeta(1-\pi_{02})}{\pi_{01}}\right), \\ (\pi_0\zeta^{\varepsilon} + (1-\pi_0))^{\frac{1}{\varepsilon}}(\pi_{02}(\psi_2 k) + (1-\pi_{02})n), & \text{if } \frac{\psi k}{n} \geq \frac{\zeta(1-\pi_{02})}{\pi_{01}}. \end{cases} \tag{23}$$

Impact of automation. As ψk goes up, it will first cross the lower threshold and eventually the upper threshold of $\zeta(1-\pi_{02})n/\pi_{01}$. From that moment onwards all human work is allocated in equilibrium to the relatively less machine-intensive sub-task 2 whereas machines are used in both sub-tasks. Thanks to the new degree of freedom – allocation of machines across sub-tasks – human and machine work are then perfectly substitutable.

Balanced growth path. In the long run, assuming that the extent of automation ψk will grow exogenously at an exponential rate g, so will grow the final output and output of each sub-task:

$$g = g_t = g_{t_1} = g_{t_2}. \tag{24}$$

Hence, full automation unpins economic growth from the capacity of human workers and instead pins it to the economy's capacity to accumulate programmable machines. As human work is no longer a growth bottleneck, the relative share of machines in generating output will grow exponentially, too ($g_\Pi = g$), and with a fixed wage rate the human labor share of output will decline, eventually to zero as $\psi k \to \infty$. The fraction of machines allocated to sub-task 1 will gradually decline from 1 to the fixed limit

$$\lim_{\psi k \to \infty} \frac{\psi_1}{\psi} = \frac{\zeta\pi_{02}}{\pi_{01} + \zeta\pi_{02}}. \tag{25}$$

As humans and machines will eventually become perfect substitutes, the human input will no longer be essential for production.

2.3 Results under Imperfect Substitutability

Let me now relax the assumption of perfect substitutability of people and machines within each sub-task, so that now $\theta \in (0,1)$. This case excludes corner solutions, so there will be smooth transitions instead of discrete jumps. However, output at the level of the whole task in equilibrium will no longer follow a CES function and the derived formulas will be somewhat less transparent.

2.3.1 Partial Automation: Sub-Task 2 Not Automatable

From wage equalization across sub-tasks (6) we get:

$$\frac{n_1}{n_2} = \left(\frac{\pi_0}{1-\pi_0}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\theta}} \left(\frac{t_1}{t_2}\right)^{\frac{\varepsilon-\theta}{1-\theta}}. \tag{26}$$

Equilibrium allocation. Assuming $\psi_2 = 0$ and thus $\psi_1 = \psi$, and therefore $t_1 = (\pi_{01}(\psi k)^\theta + (1-\pi_{01})n_1^\theta)^{1/\theta}$ and $t_2 = (1-\pi_{02})^{1/\theta} n_2$, I obtain that employment in sub-task 1 solves the implicit equation:

$$\left(\pi_{01}(\psi k)^\theta + (1-\pi_{01})n_1^\theta\right)^{\frac{\varepsilon-\theta}{\theta}} n_1^{\theta-1}(n-n_1)^{1-\varepsilon} = \frac{1-\pi_0}{\pi_0}\frac{(1-\pi_{02})^{\frac{\varepsilon}{\theta}}}{(1-\pi_{01})}. \tag{27}$$

As the left hand side is strictly decreasing in both n_1 and ψk, from the implicit function theorem it is easily obtained that (i) the solution to (27) is unique, and (ii) the fraction of people employed in the automatable sub-task 1 gradually declines with the extent of automation. As $\psi k \to \infty$, the equilibrium share $n_1^*(\psi k)$ must fall to zero, and in the limit all employment will eventually become concentrated in the non-automatable sector.

Factor shares and substitutability. If $\psi_2 = 0$ and thus machines are not employed in sub-task 2, the relative factor share in the economy Π equals:

$$\Pi = \frac{\pi_K}{\pi_L} = \frac{\pi\pi_1}{\pi(1-\pi_1)+(1-\pi)} = \frac{\pi_0\pi_{01}t_1^{\varepsilon-\theta}(\psi k)^\theta}{\pi_0(1-\pi_{01})t_1^{\varepsilon-\theta}n_1^\theta + (1-\pi_0)t_2^\varepsilon}. \tag{28}$$

Hence, after some algebra,

$$\Pi = \left(\frac{\psi k}{n}\right)^\theta \left(\frac{n_1}{n}\right)^{1-\theta} \frac{\pi_{01}}{1-\pi_{01}}, \tag{29}$$

where $n_1 = n_1^*(\psi k)$ is the solution to equation (27). Equation (29) reveals that as the extent of automation grows, the human share of output grows, too. For sufficiently large ψk, human and machine work become gross complements at the level of the whole task, because the exclusively human-operated sub-task 2 is complementary to the mostly machine-operated sub-task 1. The aggregate elasticity of substitution (Xue and Yip, 2013) gradually declines from a high value of $\frac{1}{1-\theta} > 1$ when $\psi k = 0$ to a low value of $\frac{1}{1-\varepsilon} < 1$ as $\psi k \to \infty$, crossing unity in the process.

It is also instructive to compute the equilibrium wage rate:

$$w = w_2 = (1-\pi)(1-\pi_2)\frac{T}{n_2 L} = (1-\pi_0)(1-\pi_{02})^{\frac{1}{\theta}}\left(\frac{t_2}{t}\right)^{\varepsilon-1}\frac{T_0}{L_0}. \tag{30}$$

Thus the wage rate is proportional to the contribution of the non-automatable sub-task 2 to overall output, t_2/t.

Impact of automation. As ψk goes up, human work is increasingly allocated to the non-automatable sub-task 2. From a certain moment onwards, human and machine work become gross complements (with an aggregate elasticity of substitution converging to the low value of $\frac{1}{1-\varepsilon} < 1$ as $\psi k \to \infty$). The human labor share of output grows, eventually to unity as $\psi k \to \infty$. Wages grow in negative sync with the declining contribution of sub-task 2 to overall output (t_2/t), mirroring the increasing scarcity of human work, but eventually converge to a firm upper bound.

Long-run steady state. In the long run steady state (in which $\psi k \to \infty$), all human work is allocated to the non-automatable sub-task 2 and the human share of output π_L is one. Output approaches the upper limit:

$$t_{\max} = (1-\pi_0)^{\frac{1}{\varepsilon}}(1-\pi_{02})^{\frac{1}{\theta}} n. \tag{31}$$

In consequence, wages approach their respective upper limit, too:

$$w_{\max} = (1-\pi_0)^{\frac{1}{\varepsilon}}(1-\pi_{02})^{\frac{1}{\theta}} \frac{T_0}{L_0}. \tag{32}$$

In the long run, human work is a bottleneck of development: total output is bounded above and further growth is impossible. The only way to circumvent this trap is to make all sub-tasks automatable, so that the human input could be no longer essential for production.

2.3.2 Full Automation: Both Sub-Tasks Automatable

When both sub-tasks are automatable, both human work and machines can be freely allocated to either of them. In an interior equilibrium, wages and rental rates of machines must be equalized across both sub-tasks (equation (16)). However, in contrast to the case $\theta = 1$, with imperfect substitutability of people and machines within sub-tasks equation (16) leads to an interior equilibrium solution with both humans and machines employed in both sub-tasks:

$$\frac{n_1}{n_2} = \frac{\psi_1}{\psi_2}\left(\frac{\pi_{02}}{\pi_{01}}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\theta}}. \tag{33}$$

Hence, both factors of production are always reallocated in unison, counteracting the complementarity between sub-tasks. We denote their ratio as $\left(\frac{\pi_{02}}{\pi_{01}}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\theta}} = \mu > 0$.

Equilibrium allocation. Assuming without loss of generality that $\pi_{01} \geq \pi_{02}$ so that sub-task 1 is relatively more machine-intensive, the allocation of workers across sub-tasks solves the implicit equation:

$$\frac{n_1}{n_2} = \left(\frac{\pi_0}{1-\pi_0}\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\theta}}\left(\frac{t_1}{t_2}\right)^{\frac{\varepsilon-\theta}{1-\theta}}, \tag{34}$$

where

$$\frac{t_1}{t_2} = \left(\frac{\pi_{01}(\psi_1 k)^\theta + (1-\pi_{01})n_1^\theta}{\pi_{02}(\psi_2 k)^\theta + (1-\pi_{02})n_2^\theta}\right)^{\frac{1}{\theta}} = \frac{n_1}{n_2}\left(\frac{\pi_{01}\left(\frac{\psi_2 k}{\mu n_2}\right)^\theta + (1-\pi_{01})}{\pi_{02}\left(\frac{\psi_2 k}{n_2}\right)^\theta + (1-\pi_{02})}\right)^{\frac{1}{\theta}} \tag{35}$$

and

$$\frac{\psi_2 k}{n_2} = \frac{\psi k}{n}\frac{\frac{n_1}{n_2}+1}{\frac{n_1}{\mu n_2}+1}. \tag{36}$$

Application of the implicit function theorem implies that (i) a unique solution $n_1^*(\psi k)$ exists, and (ii) as long as $\pi_{01} > \pi_{02}$, allocation of human work in the first (more machine-intensive) sub-task $n_1^*(\psi k)$ declines with ψk. As $\psi k \to \infty$, n_1^* converges from above to a positive constant. With $n_1^*(\psi k)$ in hand, the allocation of machines $\psi_1^*(\psi k)$ is calculated from (33).

The particular case $\pi_{01} = \pi_{02}$ implies $\mu = 1$. The division of factors across sub-tasks then does not depend on relative factor endowments:

$$\frac{\psi_1}{n_1} = \frac{\psi_2}{n_2} = \frac{\psi}{n}, \qquad \frac{t_1}{t_2} = \frac{n_1}{n_2} = \frac{\psi_1}{\psi_2} = \left(\frac{\pi_0}{1-\pi_0}\right)^{\frac{1}{1-\varepsilon}}, \tag{37}$$

In such a case, the problem simplifies greatly and the aggregate production function retains the normalized CES form with a high elasticity of substitution $\frac{1}{1-\theta} > 1$:

$$t = (1-\pi_0)^{\frac{1}{\varepsilon}}\left(\left(\frac{\pi_0}{1-\pi_0}\right)^{\frac{1}{1-\varepsilon}} + 1\right)^{\frac{1-\varepsilon}{\varepsilon}}\left(\pi_{01}(\psi k)^\theta + (1-\pi_{01})n^\theta\right)^{\frac{1}{1-\theta}}. \tag{38}$$

If $\psi_{01} \neq \psi_{02}$ then a closed form of the aggregate production function cannot be obtained.

As the extent of automation grows, in the case $\pi_{01} > \pi_{02}$ (where sub-task 1 is relatively more machine-intensive) human work is gradually reallocated towards sub-task 2, and machines – towards sub-task 1. In the case $\pi_{01} = \pi_{02}$, the division of factors between sub-tasks is fixed. In both cases reallocation of factors across sub-tasks helps circumvent the fact that the sub-tasks are mutually complementary. In result, the high degree of substitutability between people and machines is passed from the level of sub-tasks to the level of the entire task.

Factor shares and substitutability. When both tasks are automatable, the relative factor share in the economy Π equals:

$$\Pi = \frac{\pi_K}{\pi_L} = \frac{\pi\pi_1 + (1-\pi)\pi_2}{\pi(1-\pi_1) + (1-\pi)(1-\pi_2)} \tag{39}$$

$$= \frac{\pi_0\pi_{01}t_1^{\varepsilon-\theta}(\psi_1 k)^\theta + (1-\pi_0)\pi_{02}t_2^{\varepsilon-\theta}(\psi_2 k)^\theta}{\pi_0(1-\pi_{01})t_1^{\varepsilon-\theta}n_1^\theta + (1-\pi_0)(1-\pi_{02})t_2^{\varepsilon-\theta}n_2^\theta}. \tag{40}$$

Hence, after some algebra,

$$\Pi = \left(\frac{\psi k}{n}\right)^{\theta} \left(\frac{1+\frac{n_1}{n_2}}{\mu+\frac{n_1}{n_2}}\right)^{\theta} \left(\frac{\frac{n_1}{n_2}\frac{\pi_{01}}{1-\pi_{01}} + \frac{\pi_{02}}{1-\pi_{02}}\mu^{\theta}}{\frac{n_1}{n_2}+1}\right), \tag{41}$$

where n_1/n_2 solves equation (34). If also $\pi_{01} = \pi_{02}$, equation (41) simplifies to:

$$\Pi = \left(\frac{\psi k}{n}\right)^{\theta} \frac{\pi_{01}}{1-\pi_{01}}. \tag{42}$$

It means that when both tasks are automatable, human and machine work are highly substitutable at the level of the whole task. The complementarity between both sub-tasks is counteracted by reallocating the factors accordingly.

The equilibrium wage rate is now

$$w = w_2 = (1-\pi)(1-\pi_2)\frac{T}{n_2 L} = (1-\pi_0)(1-\pi_{02})\left(\frac{t_2}{t}\right)^{\varepsilon-1}\left(\frac{n_2}{t_2}\right)^{\theta-1}\frac{T_0}{L_0}, \tag{43}$$

and thus is shaped by two factors: (i) the contribution of sub-task 2 to overall output and (ii) the labor intensity of sub-task 2.

Impact of automation. As the extent of automation goes up, the human labor share of output declines, eventually to zero as $\psi k \to \infty$. Wages continue to grow indefinitely, albeit slower than output because of the falling labor intensity of sub-task 2. In the long run, total output grows proportionally to the productivity-adjusted stock of programmable machines, ψk.

Balanced growth path. In the long run, assuming that the per capita stock of machines ψk will grow exogenously at an exponential rate g, so will grow the final output and output of each sub-task:

$$g = g_t = g_{t_1} = g_{t_2}. \tag{44}$$

Hence, full automation unpins economic growth from the capacity of human workers and instead pins it to the stock of programmable machines. As human work is no longer a growth bottleneck, the relative share of machines in generating output will grow exponentially, too ($g_\Pi = g$), and the human labor share of output will decline, eventually to zero as $\psi k \to \infty$.

The fraction of people allocated to the respective sub-tasks will gradually converge to a finite limit:

$$\lim_{\psi k \to \infty} \frac{n_1}{n_2} = \left(\frac{\pi_0}{1-\pi_0}\right)^{\frac{1}{1-\varepsilon}} \left(\frac{1-\pi_{01}}{1-\pi_{02}}\right)^{\frac{1}{1-\theta}} \left(\frac{\pi_{01}}{\pi_{02}}\right)^{\frac{\varepsilon-\theta}{(1-\varepsilon)\theta(1-\theta)}}, \tag{45}$$

and so will the fraction of machines, $\frac{\psi_1}{\psi_2} = \frac{n_1}{\mu n_2}$, and the proportion t_1/t_2. Wages will eventually set on an exponential growth path:

$$g_w = (1-\theta)g, \tag{46}$$

mirroring the assumption that with $\theta < 1$, there is a little complementarity between human and machine inputs, and thus a part of the productivity increase due to progressing automation spills over to the workers. As $\psi k \to \infty$, overall output will become proportional to the output of each of the two tasks t_1, t_2 and the elasticity of substitution between people and machines will converge to $\frac{1}{1-\theta} > 1$, so that people and machines will be gross substitutes and the human input will no longer be essential for production.

2.4 Technological Unemployment?

In the discussion so far, there was no technological unemployment because the number of productivity-adjusted hours worked in the economy nL was considered fixed. People supplied their labor inelastically for any wage. If this assumption is relaxed, though, leading to an upward sloping labor supply curve, results change. Under the *partial automation* scenario, increases in the extent of automation increase employment in equilibrium; the *full automation* scenario, by contrast, implies rising technological unemployment.

To see this, note that in the model discussed above, under *full automation* as $\psi k \to \infty$ final output becomes proportional to output of either of the two tasks, which follow a CES production function with gross substitutability between inputs ($\theta > 0$). In particular in the case $\theta = 1$ final output is linear in the human and machine input. Under *partial automation*, in contrast, as $\psi k \to \infty$ final output is driven exclusively by the scarce non-automatable sub-task 2.

As a simple example illustrating how the mechanism works, consider the static problem of a representative household which maximizes utility from consumption and leisure subject to the constraint that all output is immediately consumed, taking ψk as given:

$$\max_{n \in [0,\bar{n})} u(c, \bar{n} - n) = \alpha \ln c + (1-\alpha) \ln(\bar{n} - n), \qquad \alpha \in (0,1), \tag{47}$$

where

$$c = t = t_0 \left(\pi (\psi k)^\theta + (1-\pi) n^\theta \right)^{\frac{1}{\theta}}, \qquad \pi \in (0,1), \theta \in (0,1], t_0 > 0. \tag{48}$$

The first order condition is

$$(1-\pi) n^{\theta-1} (\alpha \bar{n} - n) = (1-\alpha) \pi (\psi k)^\theta. \tag{49}$$

The results under partial and full automation are very different. Under *full automation*, in the linear case $\theta = 1$ we obtain an explicit solution:

$$n = \begin{cases} \alpha \bar{n} - (1-\alpha) \frac{\pi}{1-\pi} (\psi k), & \text{if } \frac{\psi k}{\bar{n}} \leq \frac{\alpha}{1-\alpha} \frac{1-\pi}{\pi}, \\ 0, & \text{if } \frac{\psi k}{\bar{n}} > \frac{\alpha}{1-\alpha} \frac{1-\pi}{\pi}. \end{cases} \tag{50}$$

Hence, when the extent of automation is sufficiently large, the equilibrium wage is too low relative to the returns on programmable machines for anyone to work. Accordingly, in the less-than-linear case $\theta \in (0, 1)$ labor supply is never quite zero, but nevertheless systematically declining with progress in automation: using the implicit function theorem it is obtained that $n^*(\psi k)$ decreases with ψk, ultimately to 0 as $\psi k \to \infty$.

Hence, under the full automation scenario with endogenous labor supply, the decline in labor demand following from progress in automation translates not only into a sub-par increase in wages (when output grows at a rate g, wages grow at a rate $(1 - \theta)g$), but also into an overall decline in employment. Full automatability of complex tasks begets technological unemployment.

This result is a polar opposite to the *partial automation* scenario, which can be easily reproduced by taking $\theta < 0$ in (48), so that in the long run machines and people are complementary, not substitutable, and human work is essential for producing final output. In such a scenario, as human work becomes increasingly scarce, $n^*(\psi k)$ firmly increases with ψk, ultimately to $\alpha\bar{n}$ as $\psi k \to \infty$.

3 Discussion and Conclusions

The current paper has discussed a new mechanism that may strongly affect our understanding of economic consequences of automation: *a shift from partial to full automatability of complex tasks*. If tasks generating value added are complex – that is, consist of at least two complementary sub-tasks – it makes a big difference if they are partially or fully automatable. The critical question in this regard is whether all sub-tasks can be automated or at least one sub-task cannot.

A shift from partial to full automatability of complex tasks is disruptive for at least four reasons. First, once a task in fully automated, people and machines switch from being complementary to substitutable in production and long-run trends in factor shares are reversed.

Second, while both partial and full automation increase inequality relative to the scenario with no automation at all, full automation does so more strongly and through a different channel. *Partial automation* leads to increases in the skill premium and polarization in the labor market (Autor and Dorn, 2013; Autor and Salomons, 2018): low- and middle-skilled routine occupations are replaced with machines and pre-programmed algorithms while high-skilled jobs complementary to the automated routine occupations thrive and increase their output share. In contrast, *full automation* leads to a declining output share of all types of human work, whether skilled or unskilled, physical or cognitive. What rises instead is the share of output accruing to (the owners of) programmable machines and their software (Barkai, 2017; Autor, Dorn, Katz, Patterson, and Van Reenen, 2017). Whether this increases inequality relative to the partial automation scenario, depends on the dispersion of high cognitive skills (which benefit most under partial automation) in the population relative to ownership of programmable machines and their software (which benefit most under full automation). In my perception, partially corroborated by the analysis by Benzell and Brynjolfsson (2019), ownership of programmable machines is likely to be more concentrated than ownership of human cognitive skills, because (i) human skills are to some extent naturally dispersed (each of us has one brain and cannot freely accumulate brainpower), (ii) in contrast, computer hardware (data processing power, data storage capacity, bandwidth) is accumulable per capita, (iii) computer software can almost costlessly scale up to the available hardware, (iv) there are increasing returns to scale in the global digital economy. If my assumptions are correct, full automation should then increase inequality more strongly than partial automation.

Third, partial automation increases the demand for human cognitive work (in complementary occupations) whereas full automation decreases it. Therefore only full, but not partial automation is conducive to technological unemployment.

Fourth, full automation undoes the bottleneck of development created by the relative scarcity of human cognitive work under partial automation. Full automation allows economic growth to decouple from the capacity of human workers and instead pins it to the stock of programmable hardware.

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