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Abstract

This paper considers the prospective sources of long-run growth in the future. Historically, in the industrial era and at the early stage of the digital era (which began approximately in the 1980s) the main growth engine is R&D. If in the future all essential production or R&D tasks will eventually be subject to automation, though, the engine of growth will be shifted to the accumulation of programmable hardware (capital), and R&D will lose its prominence. By contrast, if neither production nor R&D tasks will be fully automated, R&D will remain the main growth engine. Additional mechanisms potentially accelerating and sustaining growth are the accumulation of R&D capital (particularly important under partial automation), and hardware-augmenting technical change.

Keywords: long-run growth, factor accumulation, technical change, automation, asymptotic dynamics.

JEL codes: O30, O40.

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1 Introduction

We are at the beginning of a mass extinction and all you can talk about is money and fairy tales of eternal economic growth. How dare you!

Greta Thunberg, at UN Global Climate Action Summit in New York

Expansion in the digital sphere is now an order of magnitude faster than growth in the global capital stock and GDP: data volume, processing power and bandwidth double every 2–3 years, whereas global GDP doubles every 20–30 years. Since the 1980s "general-purpose computing capacity grew at an annual rate of 58%. The world's capacity for bidirectional telecommunication grew at 28% per year, closely followed by the increase in globally stored information (23%)" (Hilbert and López, 2011). The costs of a standard computation have been declining by 53% per year on average since 1940 (Nordhaus, 2017). The processing, storage, and communication of information has decoupled from the cognitive capacities of the human brain: "less than one percent of information was in digital format in the mid-1980s, growing to more than 99% today" (Gillings, Hilbert, and Kemp, 2016). Preliminary evidence also suggests that since the 1980s the efficiency of computer algorithms has been improving at a pace that is of the same order of magnitude as accumulation of digital hardware (Grace, 2013; Hernandez and Brown, 2020). Corroborating this finding, in the recent decade we have witnessed a surge in AI breakthroughs based on the methodology of *deep neural networks* (Tegmark, 2017), from autonomous vehicles and simultaneous language interpretation to self-taught superhuman performance at chess and Go (Silver, Hubert, Schrittwieser, et al., 2018).

However, the jury is still out on how (if at all) these tendencies will affect global long-run economic growth in the coming decades. Some economists such as Jones (2002); Gordon (2016) have documented that the trend growth rate of labor productivity and TFP has been slowing down since the 1980s and formulated a hypothesis that the global economy is therefore heading towards secular stagnation. Other economists, such as Brynjolfsson and McAfee (2014); Brynjolfsson, Rock, and Syverson (2019), put forward an alternative hypothesis that the recent slowdown in productivity growth is only temporary and represents a transition phase between the industrial and the digital era, which – not quite coincidentally – just took off in the 1980s. Once the transition phase is over, the trend growth rate in productivity, fueled by the rapidly increasing capacity of digital technologies, will rebound and perhaps even surpass the one observed in the 1950s–1980s. Beyond economics, futurists and artificial intelligence (AI) researchers have pointed out the likelihood of an upcoming technological singularity (Kurzweil, 2005; Hanson and Yudkowsky, 2013) – which would imply even greater growth acceleration.

In an attempt to formally structure this discussion, in my recent paper (Growiec, 2019) I presented the *hardware–software model*. My key proposition there was to replace capital and labor as key macroeconomic factors of production with broader aggregates: hardware and software. I started off from the basic observation that output is generated through purposefully initiated physical action. Generating output requires both some physical action – carried out by *hardware* – and some code, information describing the action – provided by *software*. This underscores that physical capital and human physical labor are fundamentally substitutable inputs, contributing to hardware; analogously, human cognitive work and pre-programmed digital software are also substitutes, contributing to software. In turn, both hardware and software are complementary and essential in the process. Furthermore, programmable hardware, such as computers, smartphones or robots, similarly to the human body has double duty: as means of performing physical actions and as a container for software – stored information and working algorithms. Technological progress represents increases the stock of available codes and thus should be generally viewed as *software-augmenting*.

In the current paper I study the prospective sources of long-run growth in the future. I formulate a range of predictions conditional on certain key assumptions regarding automatability of production and R&D tasks and structure of the R&D process. Specifically I consider the following research questions.

- **Full vs. Partial Automation.** *How are long-run growth predictions affected whether or not all essential tasks can be automated? (Growiec, 2020)*

  I find that when all essential production and R&D tasks are subject to automation, the long-run growth engine, determining the GDP growth rate in the long-run limit, is the accumulation of programmable hardware. If some essential tasks cannot be automated, though, both in production and R&D, there is a dual long-run growth engine, consisting of R&D and the accumulation of programmable hardware (used as R&D capital). Then the GDP growth rate in the long-run limit is pinned by the rate of R&D.

- **Automation of Production vs. R&D.** *How are long-run growth predictions affected if only production, but not R&D tasks can be automated? And conversely, what if only R&D, but not production tasks can be automated?*

  I find that when all essential production tasks are subject to automation, the long-run growth engine, determining the GDP growth rate in the long-run limit, is the accumulation of programmable hardware. Whether or not R&D tasks can be automated as well, is irrelevant for long-run growth. In contrast, if some essential production tasks cannot be automated but all R&D tasks can, automation of R&D has the potential of accelerating and sustaining long-run growth by creating an additional positive feedback loop. Then the long-run growth engine, determining the GDP growth rate in the long-run limit, is again the accumulation of programmable hardware.

- **R&D Capital.** *How are long-run growth predictions affected whether or not machines (physical capital) can be used in the R&D process?*

  I find that when all essential production tasks are subject to automation, presence or absence of R&D capital in the R&D process is irrelevant for the GDP growth rate in the long-run limit. If some essential tasks cannot be automated, though, both in production and R&D, accumulation of R&D capital has the potential of accelerating and sustaining long-run growth by creating an additional positive feedback loop.

- **Hardware-Augmenting Technical Change.** *How are long-run growth predictions affected whether or not technical change can be (at least partly) hardware-augmenting?*

  I find that when all essential production or R&D tasks are subject to automation, hardware-augmenting technical change leads to explosive growth with unboundedly increasing GDP growth rates. This occurs due to the creation of a self-reinforcing positive feedback loop between hardware accumulation and technical change. If some essential tasks both in production and R&D cannot be automated, though, hardware-augmenting technical change (unless very strong) becomes irrelevant for the GDP growth rate in the long-run limit.

These results are intuitive. The key variable to observe is the relatively scarce factor of production in the long-run limit. Is it hardware or software? In the pre-1980 industrial economy, where production processes were increasingly mechanized but not automated (capital was gradually replacing human physical labor in performing physical actions, but instructions for the actions were provided exclusively by people), the scarce factor was human cognitive work, which is not accumulable per capita. Then the key source of growth was labor-augmenting technological progress, provided by R&D (Romer, 1990; Jones, 1995; Acemoglu, 2009). In the post-1980 digital economy, though, as production processes get automated (instructions for physical actions are increasingly stored and run on programmable hardware), labor is replaced with capital in *software*. Automation contributes to economic growth alongside R&D, but thus far R&D remains the key growth engine because there is a wide range of tasks which – with today's technology – cannot be automated. Eventually, when all essential tasks will be automated, labor (human cognitive work) will give way to capital (programmable hardware) as the scarce factor of production. Then the key source of growth will be the accumulation of capital (Jones and Manuelli, 1990; Growiec, 2019). If, in contrast, some essential production and R&D tasks would never be automated, human labor employed in these tasks will remain the scarce factor of production, and labor-augmenting technological progress which improves their productivity will remain the key engine of long-run growth.

On top of that, the hypothetical force of hardware-augmenting technical change – understood e.g. as increases in energy efficiency of computers and other programmable machines – can alleviate the scarcity of programmable hardware. Its effects will therefore be particularly notable in the scenarios where hardware really is relatively scarce.

The paper is related more broadly to studies focusing on automation and its impacts on productivity, employment, wages and factor shares (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2018; Andrews, Criscuolo, and Gal, 2016; Arntz, Gregory, and Zierahn, 2016; Frey and Osborne, 2017; Barkai, 2017; Autor, Dorn, Katz, Patterson, and Van Reenen, 2017; Jones and Kim, 2018; Hemous and Olsen, 2018). It also touches the nascent literature on macroeconomic implications of development of "digital/robotic/machine labor", AI and autonomous robots (Yudkowsky, 2013; Graetz and Michaels, 2018; Sachs, Benzell, and LaGarda, 2015; Benzell, Kotlikoff, LaGarda, and Sachs, 2015; DeCanio, 2016; Acemoglu and Restrepo, 2018; Aghion, Jones, and Jones, 2019; Berg, Buffie, and Zanna, 2018; Benzell and Brynjolfsson, 2019).

The remainder of the paper is structured as follows. Section 2 presents the key assumptions of the hardware–software model. Section 3 deals with the role of partial vs. full automation. Section 4 covers the mixed cases where full automation is possible only in production or only in R&D. Section 5 discusses the role of R&D capital. Section 6 covers hardware-augmenting technical change. Section 7 concludes.

2 Overview of the Hardware–Software Model

The hardware–software model (Growiec, 2019) begins with the observation that all output is generated through purposefully initiated physical action. In other words, producing output requires both some physical *action* and some *code*, a set of instructions describing and purposefully initiating the action. Based on this premise I posit a general production function (for whatever output) featuring some physical *hardware* X, able to perform the action, and some disembodied *software* S, providing information on what should be done and how:

$$Output = \mathcal{F}(X, S). \tag{1}$$

I assume furthermore that $\mathcal{F}$ is increasing and concave in both factors and such that hardware X and software S are essential (i.e., $\mathcal{F}(0, S) = \mathcal{F}(X, 0) = 0$) and mutually complementary in production (the elasticity of substitution between X and S is below unity). One natural way to instantiate this assumption is to take a CES specification with an elasticity of substitution $\sigma \in (0, 1)$, cf. Klump, McAdam, and Willman (2007, 2012). The particular CES form of the $\mathcal{F}$ function is however not necessary for the results.

Hardware X includes physical actions performed by both humans and machines. Hence, X encompasses both the services of physical capital K and human physical labor L, where the latter variable excludes any know-how or skill of the worker.

Software S, in turn, encompasses all useful instructions which stem from the available information, in particular the practical implementation of state-of-the-art technologies. Hence, it includes the skills and technological knowledge employed in human cognitive work, H, as well as pre-programmed software Ψ, which is essentially a task-specific list of instructions to be performed by the associated programmable hardware (e.g., computers, robots, smartphones, etc.). Pre-programmed software Ψ may in particular include artificial intelligence (AI) algorithms, able to learn from data as well as potentially self-improve and self-replicate.

Within hardware, capital and labor are generally substitutable as agents of physical action (elasticity of substitution above unity). This reflects the idea that whatever performs a given set of actions, if the actions are the same then the outcome should be the same, too. The same logic applies to software: regardless of whether a set of instructions comes from a human brain or a mechanical information processing unit, if the actual information content of instructions is the same, then the outcome should be the same, too. A question remains, though, whether all types of instructions can be provided by both people and machines, that is whether all essential tasks can be automated. If certain essential tasks cannot be automated, the reduced form production function will feature complementarity of human cognitive work and pre-programmed software in S (elasticity of substitution below unity, see the derivation in Growiec, 2020).

Formally, I represent these assumptions as:

$$X = G_1(L, K), \qquad S = G_2(H, \Psi), \tag{2}$$

where the elasticity of substitution in G_1 is above one, and in G_2 – above one under the full automation scenario, and below one in the partial automation scenario. Each of the four factors of production has its unique properties.

- *Human physical labor* L is rivalrous and given in fixed supply per worker and unit of time, $L = \zeta N$ where $\zeta \in [0, \bar{\zeta}]$ denotes the supply of physical labor per worker in a unit of time, expressed in physical capital units, and N is the total number of workers.

- *Physical capital* K is rivalrous but can be unboundedly accumulated in per-capita terms. Physical capital K may be non-programmable or programmable. The share of programmable hardware in total physical capital is denoted by χ (so that $\chi \in [0, 1]$).

- *Human cognitive work* H consists of three components, technological knowledge A, the average skill level h, and the number of workers N, as in $H = AhN$.

Technological knowledge A, or the size of the "repository of codes" is non-rivalrous (Romer, 1986, 1990) and accumulable. Per-capita skill levels h are rivalrous and bounded above.

- *Pre-programmed software* Ψ also consists of three components, technological knowledge A, algorithmic skill level ψ which captures the degree to which pre-programmed software is able to perform the tasks collected in A, and the stock of programmable hardware χK on which the software is run, as in $\Psi = A\psi\chi K$. Technological knowledge A is the same as above.[1] The algorithmic skill level ψ is assumed to be bounded above by the optimal code for performing a given task (i.e., perfect accuracy), though there may be in fact a much lower upper bound $\bar{\psi}$ (Hanson and Yudkowsky, 2013). Because software can be virtually costlessly copied, it is assumed that it can scale up to the level of all available programmable hardware χK.

All in all, the general aggregate production function takes the form:

$$Output = \mathcal{F}(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)). \tag{3}$$

Finally, following Romer (1986, 1990), the hardware–software model envisages technological progress (growth in A) as expansion of the "repository of codes", i.e., as the development of new, better instructions allowing to produce higher output with a given amount of hardware. Whether these new instructions take the form of new abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items, they are all *information* and not actual *objects* or *actions*, and it is precisely this informational character that makes technologies non-rivalrous and a source of increasing returns to scale (Romer, 1990). In contrast to Paul Romer's seminal contributions, though, here these instructions can be applied to the tasks at hand both by humans and machines. Thus all technological progress is naturally modeled as *software-augmenting*.

In the following sections I will derive long-run growth predictions from the baseline hardware–software model and its variations. I will represent the baseline case as the following reduced-form two-sector growth model with a production and R&D sector:

$$\begin{aligned}
Y &= F(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)), && (4)\\
\dot{A} &= A^{\phi}\Phi(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)), && (5)\\
\dot{K} &= sY - \delta K, && (6)
\end{aligned}$$

where the term A^{ϕ} (with $\phi \in [0, 1]$) captures the potentially positive "standing on shoulders" effects in R&D (Jones, 1995). I will also assume that F, G_1, G_2 and Φ

[1] If in reality the sets of codes available to humans and AI are different, the discrepancy between the measures of both sets can be captured by the factor ψ relative to h.

are characterized by constant returns to scale. Finally, I will posit that bounded variables (s, h, ψ, χ) will eventually stabilize and that population N is constant. Thus I will concentrate solely on the dynamics of two state variables of the model, K and A, in the long-run limit.

In the following analysis I will use the following asymptotic notation:

$$a_K = F(1,\infty) = \lim_{y\to\infty} F(1,y), \qquad b_K = \Phi(1,\infty) = \lim_{y\to\infty} \Phi(1,y), \tag{7}$$

$$a_N = F(\infty,1) = \lim_{x\to\infty} F(x,1), \qquad b_N = \Phi(\infty,1) = \lim_{x\to\infty} \Phi(x,1). \tag{8}$$

By the assumption of elasticity of substitution in F and Φ being strictly less than unity, all these limits exist and are finite.

3 Full vs. Partial Automation

The baseline specification of the hardware–software model assumes high substitutability of human and machine inputs within hardware and software (elasticity of substitution above one in G_1 and G_2). This assumption may be violated, though, when dealing with complex, multi-step tasks that have not been yet fully automated. In Growiec (2020) I have demonstrated, using a two-level nested CES production function specification, that when a cognitive task consists of two or more necessary steps, some of which can be performed by a computer algorithm but others (under current technology) only by a human, then pre-programmed software and human cognitive work will turn out complementary at the level of the whole task even if they are substitutable within the sub-tasks. This complementarity (elasticity of substitution below one) in the reduced form production function appears only in the *partial automation* scenario; once the whole task becomes fully automatable, human cognitive work and pre-programmed software become substitutable again (elasticity of substitution above one). For this reason the scenarios of partial vs. full automation ought to be considered separately.

Full Automation of Production and R&D. In the long-run limit, as all tasks eventually get mechanized and automated, with $K/N \to \infty$, for computing the long-run dynamics we may approximate $X = G_1(\zeta N, K) \approx K$ and $S = G_2(AhN, A\psi\chi K) \approx A\psi\chi K$. As $K \to \infty$ and $A \to \infty$, asymptotically

$$Y = F(K, A\psi\chi K) = KF(1, A\psi\chi) \to a_K K, \tag{9}$$

$$\dot{A} = A^{\phi}\Phi(K, A\psi\chi K) = A^{\phi}K\Phi(1, A\psi\chi) \to b_K A^{\phi} K, \tag{10}$$

$$\dot{K} \approx (sa_K - \delta)K. \tag{11}$$

This means that when all essential tasks are subject to automation, the GDP growth rate will converge to $g = sa_K - \delta$, and the long-run growth engine will be the accumulation of programmable hardware (Jones and Manuelli, 1990). Thanks to the additional positive contribution of R&D, software will grow systematically faster than hardware, and therefore (programmable) hardware will eventually become the scarce factor of production. The pace of technical change (growth in A), while important over the transition, will eventually become irrelevant for growth.

Partial or No Automation of Production and R&D. In the long-run limit, all tasks will eventually get mechanized while a fraction of essential production and R&D tasks will forever remain immune to automation, making human cognitive work and pre-programmed software complementary (elasticity of substitution below one in G_2). As $K/N \to \infty$, for computing the long-run dynamics we may approximate $X = G_1(\zeta N, K) \approx K$ and $S = G_2(AhN, A\psi\chi K) \approx AhN$. There are two cases to consider, either $\phi = 0$ or $\phi \in (0, 1]$. If $\phi = 0$ then an asymptotical balanced growth path is attained as $K \to \infty$ and $A \to \infty$, with

$$g = g_A = \Phi\left(\frac{K}{A}, hN\right) = sF\left(1, \frac{A}{K}hN\right) - \delta. \tag{12}$$

In this case, the long-run GDP growth rate is determined by the pace of R&D, which is in turn sustained by the accumulation of R&D capital. This is a dual growth engine, and both hardware and software grow asymptotically at the same rate g.

In contrast, with positive R&D spillovers ("standing on shoulders", Jones, 1995), asymptotically A will grow faster than K. When $\phi \in (0, 1]$, in the long-run limit

$$\begin{aligned} Y &= F(K, AhN) = KF\left(1, \frac{A}{K}hN\right) \to a_K K, && (13)\\ \dot{A} &= A^{\phi}\Phi(K, AhN) = A^{\phi}K\Phi\left(1, \frac{A}{K}hN\right) \to b_K A^{\phi} K, && (14)\\ \dot{K} &\approx (sa_K - \delta)K. && (15) \end{aligned}$$

Hence, in the presence of R&D capital accumulation, positive R&D spillovers reinstate the Jones and Manuelli (1990) dynamic even if production and R&D tasks are only partially automatable or not at all. The long-run growth engine is then again accumulation of programmable hardware, setting the GDP growth rate in the long-run limit as $g = sa_K - \delta$.

4 Full Automation Only in Production or R&D

Existing literature suggests that routine tasks, both manual and cognitive, are relatively easiest to automate, while automation gets harder for tasks which are more complex and carried out in a less structured environment. Among all tasks, cutting-edge R&D tasks requiring sophisticated reasoning and out-of-the-box thinking are probably among the least susceptible to automation (Acemoglu and Autor, 2011; Autor and Dorn, 2013; Frey and Osborne, 2017; Acemoglu and Restrepo, 2018). It is therefore natural to expect that production tasks may – if at all – become fully automatable earlier than R&D tasks. On the other hand, AI algorithms are already entering some more routine research tasks, like scanning astronomical photographs or sequencing genomes, while some seemingly easy motor tasks remain notoriously difficult to automate – so perhaps we should be cautious with such predictions.

In order not to miss any viable scenario of the future, in the following paragraphs I discuss two polar scenarios: one in which production eventually becomes fully automatable, whereas R&D does not (Acemoglu and Restrepo, 2018), and the one where eventually R&D becomes fully automatable, whereas production does not.

Full Automation Only in Production. In the long-run limit, as all production tasks eventually get mechanized and automated while human cognitive work remains essential for R&D, asymptotically (with $K \to \infty$ and $A \to \infty$) I obtain:

$$Y \approx F(K, A\psi\chi K) = KF(1, A\psi\chi) \to a_K K, \tag{16}$$

$$\dot{A} \approx A^{\phi}\Phi(K, AhN), \tag{17}$$

$$\dot{K} \approx (sa_K - \delta)K. \tag{18}$$

Hence, once all essential production tasks will be automated, the long-run GDP growth rate will converge to $g = sa_K - \delta$, and the long-run growth engine will be the accumulation of programmable hardware (Jones and Manuelli, 1990). Thanks to the additional positive contribution of R&D, software will grow systematically faster than hardware, and therefore (programmable) hardware will eventually become the scarce factor of production. Along the transition, the pace of technical change (growth in A) will be markedly lower than in the scenario where R&D tasks are automated as well. This will drag also on GDP growth. In the long-run limit, though, the role of R&D will vanish and its pace will eventually become irrelevant for the pace of GDP growth.

Full Automation Only in R&D. In the long-run limit, as all R&D tasks will eventually get mechanized and automated while human cognitive work will remain essential for production, asymptotically A will be growing faster than K, implying (as $A \to \infty$ and $K \to \infty$):

$$Y \approx F(K, AhN) = KF\left(1, \frac{A}{K}hN\right) \to a_K K, \tag{19}$$

$$\dot{A} \approx A^{\phi}\Phi(K, A\psi\chi K) = A^{\phi}K\Phi\left(1, A\psi\chi\right) \to b_K A^{\phi} K, \tag{20}$$

$$\dot{K} = (sa_K - \delta)K. \tag{21}$$

It turns out that full automation of R&D tasks is sufficient for generating the Jones and Manuelli (1990) dynamic with a long-run GDP growth rate $g = sa_K - \delta$

even if production tasks are only partially automatable or not at all. The long-run growth engine is then again accumulation of programmable hardware. Compared to the scenario where neither production nor R&D tasks are fully automatable, full automation of R&D creates an additional positive feedback loop, accelerating and sustaining long-run growth. Compared to the scenario with full automation in production and R&D, though, the GDP growth rate is markedly lower over the transition and only converges to the same one in the long-run limit.

5 R&D Capital

Nowadays R&D processes increasingly use sophisticated machinery. 21st century science would not be possible without sophisticated lab equipment, not to mention the personal computers on researchers' laps. Growth theory thus far has however rarely acknowledged this fact, focusing on the other crucial R&D input – researchers' skilled efforts. Hence, to bring the current paper closer to the established R&D-based growth literature (Romer, 1990; Jones, 1995; Barro and Sala-i-Martin, 2003; Acemoglu, 2009), I will now ask the question if the long-run predictions of the hardware–software model would be affected if the role of R&D capital were disregarded. Therefore in the following paragraphs I consider a version of the hardware–software model without R&D capital, i.e., without allowing R&D processes to be mechanized. To this end I fix $X = \zeta N$ in the R&D sector. I separately discuss the cases of partial vs. full automation in production and R&D.

No R&D Capital, Full Automation in Production. In the long-run limit, as all production tasks eventually get mechanized and automated ($X \approx K, S \approx A\psi\chi K$) while human physical work remains essential for R&D tasks ($X = \zeta N, S \approx A\psi\chi K$), asymptotically I obtain as $A \to \infty$ and $K \to \infty$:

$$Y = F(K, A\psi\chi K) = KF(1, A\psi\chi) \to a_K K, \tag{22}$$

$$\dot{A} = A^\phi \Phi(\zeta N, A\psi\chi K) = A^\phi \zeta N \Phi\left(1, A\frac{\psi\chi K}{\zeta N}\right) \to b_K A^\phi \zeta N, \tag{23}$$

$$\dot{K} \approx (sa_K - \delta)K. \tag{24}$$

Alternatively, with partial or no automation of R&D tasks ($S \approx AhN$ in R&D), the R&D equation becomes $\dot{A} = A^\phi \Phi(\zeta N, AhN) = A^\phi \zeta N \Phi\left(1, A\frac{h}{\zeta}\right) \to b_K A^\phi \zeta N$, with exactly the same asymptotic result.

Hence, I observe that with full automation of production tasks, the accumulation of programmable hardware is the key growth engine over the long run, and the GDP growth rate converges to $g = sa_K - \delta$ in the limit. While important over the transition, under full automation of production the presence or absence of R&D capital in the R&D process is irrelevant for the asymptotic results.

No R&D Capital, Full Automation Only in R&D. In the long-run limit, as all production tasks eventually get mechanized but not automated ($X \approx K, S \approx AhN$) while human physical but not cognitive work remains essential for R&D tasks ($X = \zeta N, S \approx A\psi\chi K$), asymptotically ($A \to \infty, K \to \infty$) A grows at a faster rate than K, implying:

$$Y = F(K, AhN) = KF\left(1, \frac{A}{K}hN\right) \to a_K K, \tag{25}$$

$$\dot{A} = A^{\phi}\Phi(\zeta N, A\psi\chi K) = A^{\phi}\zeta N\Phi\left(1, A\frac{\psi\chi K}{\zeta N}\right) \to b_K A^{\phi}\zeta N, \tag{26}$$

$$\dot{K} \approx (sa_K - \delta)K. \tag{27}$$

It turns out that full automation of R&D tasks suffices to make accumulation of programmable hardware the key growth engine over the long run. Asymptotically, the economy follows the Jones and Manuelli (1990) dynamic and the GDP growth rate converges to $g = sa_K - \delta$. In the short to medium run, though, the failure to fully automate production processes provides a clear and major drag on the pace of growth.

No R&D Capital, Partial or No Automation. In the long-run limit, as all tasks eventually get mechanized but a fraction of essential production and R&D tasks is immune to automation (Growiec, 2020), for computing the long-run dynamics (where $K/N \to \infty$) we may approximate $X \approx K$ in production and $S \approx AhN$ in production and R&D. For the latter sector we consequently obtain as $A \to \infty$ and $K \to \infty$:

$$\dot{A} = A^{\phi}\Phi(\zeta N, AhN) = A^{\phi}\zeta N\Phi\left(1, A\frac{h}{\zeta}\right) \to b_K A^{\phi}\zeta N. \tag{28}$$

Hence, with constant population N technology progresses sub-exponentially if $\phi \in [0, 1)$ (Jones, 1995; Groth, Koch, and Steger, 2010), or exponentially if $\phi = 1$ (Romer, 1990). In the absence of R&D capital and automation, the hardware–software model reproduces long known scenarios of R&D based growth, either semi-endogenous ($\phi < 1$, Jones) or fully endogenous ($\phi = 1$, Romer). The ultimate source of growth, determining the long-run GDP growth rate, is R&D. In the absence of automation, software (in this case synonymous with human cognitive work) forever remains the scarce factor of production.

Specifically in the linear case of $\phi = 1$, the GDP growth rate converges asymptotically to:

$$g_A = g_K = g_Y = b_K \zeta N, \tag{29}$$

and thus is proportional to the "weakest link" in the economy, unaugmentable physical labor employed in R&D. If $\phi < 1$, without population growth the R&D rate, and consequently the GDP growth rate, is bound to systematically slow down over time (Jones, 1995), in line with the secular stagnation scenario.

6 Hardware-Augmenting Technical Change

In Growiec (2019) I have argued, grounding my points in Romer's seminal contributions, that technical change should be generally modeled as software-augmenting. After all, technological progress (growth in A) represents expansions of the "repository of codes", i.e., the development of new, better instructions allowing to produce higher output with a given amount of hardware. These instructions are *information* and not actual objects or actions, and it is precisely this informational character that makes technologies non-rivalrous and a source of increasing returns to scale (Romer, 1986, 1990).

Hardware, by contrast, performs physical actions which require expediting energy. With this in mind, even if going against the spirit of Romer's theory, it may be conjectured that certain improvements in energy efficiency of physical actions could potentially count as hardware-augmenting. In the following paragraphs I will entertain this possibility.

At this point, it is critical to mark that hardware-augmenting technical change can resolve the scarcity of programmable hardware in production in the long run limit, thereby accelerating growth beyond the pace determined by the pace of capital accumulation. When accumulation of programmable hardware is accompanied with R&D which is at least partly hardware-augmenting, these two forces have the potential of mutual reinforcement, and thus created self-reinforcing feedback loops may lead to explosive, super-exponential growth.

Technically, the key modification of the framework is that the hardware factor in production and R&D is now technologically augmented: $X = G_1(A^\kappa(\zeta N + K), A(hN + \psi\chi K))$, with $\kappa \in (0, 1)$ representing the assumption that technological progress remains biased towards software. Furthermore, to make an even stronger case for the importance of hardware-augmenting technical change, I will ignore possible positive R&D spillovers ("standing on shoulders" effects) by setting $\phi = 0$. The remaining assumptions remain in place. The results are as follows.

Hardware-Augmenting Technical Change with Full Automation. In the long-run limit, as all tasks will eventually get mechanized and automated, for computing the long-run dynamics we may approximate $X \approx A^\kappa K$ and $S \approx A\psi\chi K$. As $A \to \infty$ and $K \to \infty$, asymptotically:

$$\begin{aligned} Y &= F(A^\kappa K, A\psi\chi K) = A^\kappa K F(1, A^{1-\kappa}\psi\chi) \to a_K A^\kappa K, && (30)\\ \dot{A} &= \Phi(A^\kappa K, A\psi\chi K) = A^\kappa K \Phi(1, A^{1-\kappa}\psi\chi) \to b_K A^\kappa K, && (31)\\ \dot{K} &\approx (s a_K A^\kappa - \delta)K. && (32)\end{aligned}$$

When essential production tasks are subject to automation and technical change is partly hardware-augmenting, the long-run GDP growth rate $g = s a_K A^\kappa - \delta$ is ever increasing over time. The dual long-run growth engine – the accumulation of programmable hardware and hardware-augmenting technical change – generates explosive growth with an unbounded growth rate.

Quick algebra proves that this result remains intact also when automation happens only in production and/or there is no R&D capital.

Hardware-Augmenting Technical Change with Partial or No Automation. In the long-run limit, as all tasks will eventually get mechanized but a fraction of essential production and R&D tasks will remain immune to automation (Growiec, 2020), for computing the dynamics we may approximate $X \approx A^\kappa K$ and $S \approx AhN$.

I find that as $A \to \infty$ and $K \to \infty$, asymptotically K grows faster than $A^{1-\kappa}$. It follows that:

$$
\begin{aligned}
Y &= F(A^\kappa K, AhN) = AhNF\left(\frac{K}{A^{1-\kappa}hN}, 1\right) \to a_N AhN, && (33)\\
\dot{A} &= \Phi(A^\kappa K, AhN) = AhN\Phi\left(\frac{K}{A^{1-\kappa}hN}, 1\right) \to b_N AhN, && (34)\\
\dot{K} &\approx s a_N AhN - \delta K. && (35)
\end{aligned}
$$

The economy converges to a balanced growth path where the long-run GDP growth rate is determined by the pace of technical change, in turn set by the human capital employed in R&D à la Romer:

$$g_A = g_K = g_Y = b_N hN. \tag{36}$$

When hardware-augmenting technical change, coupled with the accumulation of R&D capital, resolves the scarcity of hardware, the scarce factor of production is software (human cognitive work). In such circumstances, the pace of hardware-augmenting technical change is no longer relevant for the long-run GDP growth rate.

If there were also positive R&D spillovers ("standing on shoulders" effects) on top of hardware-augmenting technical change, though ($\phi \in (0, 1]$), then the R&D equation would have been explosive. To see this, take

$$\dot{A} = A^\phi \Phi(A^\kappa K, AhN) = A^{1+\phi} hN\Phi\left(\frac{K}{A^{1-\kappa}hN}, 1\right) \to b_N A^{1+\phi} hN. \tag{37}$$

This equation implies an ever increasing growth rate of technology, $g_A = A^\phi b_N hN$, which – given that output Y is proportional to A – implies explosive growth.

Hardware-Augmenting Technical Change with Partial or No Automation and No R&D Capital. Let us now check how potent hardware-augmenting technical change is for generating long-run growth under the relatively most adverse circumstance: when there is partial or no automation and no R&D capital. In the long-run limit, there will be full mechanization in production. With $K \to \infty$ and $A \to \infty$ I obtain that K grows faster than $A^{1-\kappa}$ and thus:

$$
\begin{aligned}
Y &\approx F(A^\kappa K, AhN) = AhNF\left(\frac{K}{A^{1-\kappa}hN}, 1\right) \to a_N AhN, \qquad (38)\\
\dot{A} &\approx \Phi(A^\kappa \zeta N, AhN) = A^\kappa \zeta N \Phi\left(1, \frac{hA^{1-\kappa}}{\zeta}\right) \to b_K A^\kappa \zeta N, \qquad (39)\\
\dot{K} &\approx sa_N AhN - \delta K. \qquad (40)
\end{aligned}
$$

In this scenario, due to $\kappa < 1$ technology A grows sub-exponentially (Jones, 1995; Groth, Koch, and Steger, 2010), and so does capital and output. The pace of hardware-augmenting technical change, while important over the transition, becomes irrelevant for the GDP growth rate in the long-run limit.

If there were also sufficiently strong R&D spillovers ("standing on shoulders" effects) on top of hardware-augmenting technical change, though ($\phi > 1 - \kappa$), then the R&D equation would have been explosive. To see this, take

$$
\dot{A} = A^\phi \Phi(A^\kappa \zeta N, AhN) = A^{\phi+\kappa} \zeta N \Phi\left(1, A^{1-\kappa}\frac{h}{\zeta}, 1\right) \to b_K A^{\phi+\kappa} \zeta N. \qquad (41)
$$

This equation implies an ever increasing growth rate of technology, $g_A = A^{\phi+\kappa-1} b_K \zeta N$, which – given that output Y is proportional to A – implies explosive growth. In the limiting knife-edge case $\phi + \kappa = 1$, growth is exponential and the GDP growth rate eventually converges to $g = b_K \zeta N$, determined by the pace of R&D. If $\phi < 1 - \kappa$, the aforementioned sub-exponential growth result remains intact.

7 Conclusion

In the current paper I have studied the prospective sources of long-run growth in the future. I have formulated a range of predictions conditional on certain key assumptions regarding automatability of production and R&D tasks and structure of the R&D process. The results follow from observing the dynamics of the relatively scarce factor of production in the long-run limit. When the scarce factor is human cognitive work, which is not accumulable per capita, then the key source of growth is labor-augmenting technological progress, provided by R&D. However, when all essential tasks are subject to automation, then labor (human cognitive work) will eventually give way to capital (programmable hardware) as the scarce factor of production. Then the key source of growth will be the accumulation of capital (Jones and Manuelli, 1990; Growiec, 2019). If, in contrast, some essential production and R&D tasks would never be automated, human labor employed in these tasks will remain the scarce factor of production, and labor-augmenting technological progress which improves their productivity will remain the key engine of long-run growth.

This study draws the span of potential long-run growth outcomes in a digital economy where production and R&D processes can be potentially automated. What remains for further research is a quantitative assessment of relative importance of the considered mechanisms. How long is the long run? How long is the transition period going to be? At which point will we realize that human cognitive work and pre-programmed software, now complementary because many tasks require the human input, have already become broadly substitutable? When will – if at all – the accumulation of programmable hardware overtake R&D as the key engine of growth? And what will be the role of accumulation of R&D capital?

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