Backward and forward linkages based on an input-output analysis – comparative study of Poland and selected European countries

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Abstract

Contemporary input–output tables provide a complete picture of the economic relationships between industries in a single economy for a given year. The analysis of these interconnections allows comparisons to be made between the structures of production in different countries. Comparisons of the strengths of backward and forward linkages between industries in a national economy provide one method for identifying “key” or “leading” sectors in that economy. The main purpose of the paper is to investigate the structure of production in the Polish economy by using backward and forward linkages and to compare this structure to the structure of production of selected European countries: Austria, Bulgaria, Belgium, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Netherlands, Norway, Slovakia, Slovenia, Sweden and United Kingdom. The data used in the study come from the symmetrical input–output tables for the 2010 year, provided by Eurostat. The comparative analysis revealed that the structure of production differs among the countries. The differences concern the identified key industries, the strength of linkages between industries and in effect the economic landscape of the country.

Keywords: input-output analysis, backward and forward linkages, multiplier product matrix, structure of production, economic landscape

1. Introduction

The framework of input–output analysis was developed by Wassily Leontief in the late 1930s, and its main purpose is to encompass the inter-connections between sectors of the economy during the production process. Further developments to the original Leontief model and the standardization of the national economic accounts under the direction of Richard Stone extended the practical application of input–output analysis. Both scientists received the Nobel Prize in Economics in recognition of their achievements – Leontief in 1973; Stone in 1984.

Contemporary input–output tables provide a complete picture of the economic relationships between industries in a single economy for a given year. The analysis of these interconnections allows comparisons to be made between the structures of production in different countries. If data are available for more than one time period, the evolution of these interconnections can also be studied. For these reasons, the input–output model has become a widely applied tool of economic analysis.

In the framework of an input–output analysis, production by a particular industry (sector) has two kinds of economic effects on other sectors in the economy. The first concerns the connection of the industry with its suppliers. If industry $j$ increases its output, it will increase its demands on the other sectors whose goods are used as inputs to production in $j$. This effect is known as backward linkage and shows the direction of causation in the usual demand-side model. The second refers to the connection of the industry with its clients. The increased output in industry $j$ means that additional amounts of its products are available to be used as inputs to other sectors for their own production – there will be increased supplies from sector $j$ for the sectors that use its goods in their production. The term forward linkage is used to indicate this kind of interconnection, and it shows the direction of causation in the supply-side model. Comparisons of the strengths of backward and forward linkages for the industries
in a national economy provide one method for identifying “key” or “leading” sectors in that economy.

The main purpose of the presented paper is to investigate the structure of production in the Polish economy by using backward and forward linkages based on the input–output framework and to compare this structure to the structure of production of selected European countries. The data used in the study come from the symmetrical input–output tables for the 2010 year.

2. Literature background
The contemporary input–output analysis framework has been developed on the fundamental works of Leontief (Leontief, 1936; Leontief et al., 1953; Leontief, 1974). Leontief himself recognized that the input–output model is an analytical formalization of the basic concepts set by the French economist Francois Quesnay (1759) about two centuries ago, which depicted income flows between economic sectors. Hereby, the linkage concept is not new in economics. Nevertheless, it is considered that the first detailed studies of inter-industry linkages based on the Leontief input–output model appeared in the late 1950s, and their main purpose was to identify the key industries that are crucial for economic development. The first attempt to identify and measure the inter-industry linkages was made by Hirschman (1958). His opinion that inter-industry linkages are one of the important factors of economic development is founded on the observation that ongoing activities induce economic agents to start up new activities. A year before Hirschman’s classical work had been published, Danish economist Rasmussen (1957) defined two linkage measures: “power of dispersion”, interpreted as a measure of backward linkage; and “sensitivity of dispersion”, i.e. a measure of forward linkage. Over the years, many refinements of these measures have been proposed and used in empirical studies. The connections between forward and backward linkages were investigated by Jones (1976). Cuello et al. (1992) incorporated additional information in order to obtain more precise measures of the economic importance of the key industries. The comparison and discussion of input–output based measures was provided by Drejer (2002). Detailed description different approaches adopted in order to offset the shortcomings of the traditional measures of linkages was presented by Miller and Blair (2009). Backward and forward linkages were used by Sonis et al. (2000) to calculate a multiplier product matrix (MPM) and to visualize the “economic landscape” of the economy.

3. Methodology and data
Leontief model
In the input–output model, the total output of an industry is equal to the sum of inter-industry demands and the final demand by the ultimate consumers of its goods and services. The total output $x_i$ of an industry $i$ is expressed as follows:

$$x_i = z_{i1} + z_{i2} + \ldots + z_{ij} + \ldots + z_{in} + y_i, \quad i = 1, 2, \ldots, n$$  \hspace{1cm} (1)

where

- $x_i$ is the total output of industry $i$,
- $z_{ij}$ is the intermediate demand of the products or services of industry $i$ by industry $j$ (i.e. the flow from industry $i$ to industry $j$),
- $y_i$ is the final demand of the products or the services of industry $i$.

The ratio of the intermediate inter-industry demand $z_{ij}$ to the total output of industry $j$ is called a technical coefficient (direct requirement coefficient) and is denoted as:

$$a_{ij} = \frac{z_{ij}}{x_j},$$  \hspace{1cm} (2)
After substituting (2) in (1), the total output of the industry \(i\) is:
\[
x_i = a_{ii}x_i + a_{i2}x_2 + ... + a_{ij}x_j + ... + a_{in}x_n + y_i, \quad i=1,\ldots,n.
\]
In matrix notation the total output of the economy is given by:
\[
X = AX + Y, \quad (3)
\]
where
- \(X\) is the column vector of industrial outputs,
- \(A\) is the square matrix of technical coefficients (direct requirement matrix),
- \(Y\) is the column vector of industrial final demands.

After rearrangement, the Leontief model is as follows:

\[
Y = (I - A)X \quad \text{or} \quad Y = LX \quad (4)
\]
where the matrix \(L = I - A\) is called the Leontief matrix.

If \(|I - A| \neq 0\), then the equation (4) can be written as:
\[
X = L^{-1}Y. \quad (5)
\]
Matrix \(L^{-1}\) is known as the Leontief inverse or the total requirements matrix. Its \(i,j\)-th element represents the sensitivity of the \(i\)-th sector total output to the 1 unit change of \(j\)-th sector final demand. In other words, matrix \(L^{-1}\) translate final demand changes into total output changes, encompassing direct and indirect effects, which are the sense of the multipliers.

**Backward and forward linkages**

In its simplest form backward and forward linkages can be derived from the direct requirement matrix \(A\). The sum of the elements in the \(j\)-th column of the matrix \(A\) is a measure of the strength of the backward linkage of sector \(j\), showing the amount by which sector \(j\) production depends on the inputs from other industries. On the other hand the sum of the elements in the \(i\)-th row of the matrix \(A\) is a measure of the strength of the forward linkage of sector \(i\), showing the amount by which sector \(i\) production is used by other industries. Since the coefficients in \(A\) capture the direct effects only, these measures are called direct backward and direct forward linkages. To encompass both direct and indirect effects, the Leontief inverse matrix (the total requirements matrix) is used.

Column sums of the Leontief inverse matrix are proposed as a total backward linkage measures. These measures are also known as output multipliers. For sector \(j\) it is given by:
\[
BL_j = \sum_{i=1}^{n} l_{ij}, \quad (6)
\]
where \(l_{ij}\) is \(i,j\) element of the Leontief inverse matrix.

The sector with the biggest backward linkage is important, because the increase in final demand of its goods by 1 unit will result in the biggest increase in total output of the economy. The most backward-linked industries provide the biggest demand-pull effects on the economy.

Similarly, row sums of the Leontief inverse matrix are proposed as a total forward linkage measure. These measures are known as well as input multipliers. For sector \(i\) it is given by:
\[
FL_i = \sum_{j=1}^{n} l_{ij}. \quad (7)
\]

The sector with the biggest forward linkage is important, because if the final demand of all industries’ goods increases by 1 unit, it will be needed (or will result in) the biggest increase
in total output of this sector. So, the most forward-linked industries provide the biggest supply-push effects on the economy.

One of the deficiencies of measures (6) and (7) is that they do not take into account the weight of the sectors’ transactions in total transactions (the relative importance of the sector). Another problem is that these measures are not normalized. To avoid such deficiencies, many approaches have been suggested in the literature (Miller and Blair, 1995). Cuello et al. proposed to weight the elements of the Leontief inverse matrix in two ways: by the proportion of the industry’s final demand to total final demand in the economy or by the proportion of the industry’s total output to total output in the economy. Using different approaches, the hierarchy of importance of the industries may alter.

In the presented study, firstly the weights are used to take into account the relative share of the industry’s intermediate demand in the total intermediate demand and after that Rasmussen’s normalization of the measures is applied.

The weights are calculated as follows:

\[
w_{ij} = \frac{z_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} z_{ij}}.\]

(8)

The emphasis of this study is to investigate the structure of production and the interindustry connections, therefore the weights used are the proportion of sector’s intermediate transactions to total intermediate transactions.

The weighted Leontief inverse matrix \( L^w \) is obtained as:

\[
L^w = L^{-1} W
\]

(9)

where \( W \) is the matrix of weights.

After backward and forward linkages are calculated from the matrix \( L^w \) as column and row sums, they are normalized as follows:

\[
BL^w_i = \frac{\sum_{j=1}^{n} l_{ij}}{\frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{n} l_{ij}}
\]

(10)

\[
FL^w_i = \frac{\sum_{j=1}^{n} l_{ij}}{\frac{1}{n} \sum_{j=1}^{n} \sum_{i=1}^{n} l_{ij}}.
\]

(11)

**Multiplier Product Matrix**

The next step is to calculate **Multiplier Product Matrix (MPM)**. MPM provides a quantitative measure of the relationships among industries and is a useful tool for graphical presentation of the economic landscape of a country. It allows to compare the economic landscapes of different countries. The \( i,j \) element of the MPM is obtained using backward and forward linkages in the following way:

\[
m_{ij} = FL^w_i \cdot BL^w_j.
\]

(12)

If there is no any linkages between the industries (hypothetical situation) the economic landscape is flat. The same is when there are interconnections, but they are symmetric – products of the FL and BL are constant. This situation is possible, when the industries are interrelated, but there is no explicit key industries. It could be said that the economy sectors
develops evenly and rely on each other in a similar pattern. The more realistic is the situation when there are some “leading” industries, and the economic landscape is not flat. In order to identify the key industries, the industries are organized into a rank-size hierarchy.

At the last stage of the study the landscapes of the countries are compared by the Euclidean distance.

Data

The following countries have been chosen for the comparative study: Austria, Bulgaria, Belgium, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Italy, Netherlands, Norway, Slovakia, Slovenia, Sweden and United Kingdom. Additionally, available data for EU27 are used to obtain similar analysis for the European Union. The data source are the symmetric input-output tables for the 2010 year, presented by Eurostat. In the tables there are about 65 industries, so the aggregation to 19 industries is made.

4. Results

After aggregation of the data, backward and forward linkages are calculated from the Leontief inverse matrix, weighted by the relative importance of an industry’s intermediate demand in total intermediated demand, and next normalized.

Figure 1 provides a juxtaposition of backward and forward linkages for all sectors of the Polish economy, set in order by the backward linkage strength. The most important sectors (the biggest BL and FL) are the sectors: Industrial products and Machinery industry. It is worth noticing that some of the industries only play a role of backward linkage industries e.g. Public administration, human health, education services (G) and others – of forward linkage industries - Mining and quarrying (Q).

Figure 1. Backward and forward linkages of the industries of the Polish economy

<table>
<thead>
<tr>
<th>Description of the abbreviations:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Products of agriculture, hunting, forestry, fishing</td>
</tr>
</tbody>
</table>
Similar figures for the rest of the studied countries are provided in Appendix 1, but the industries are organized in a rank-hierarchy of the EU27 backward linkages in order to provide comparability. The same hierarchy for the Polish economy is presented in figure 2.

Figure 2. Backward and forward linkages of the industries of the Polish economy (UE27 hierarchy)

Figure 3 shows the dispersion of the industries according to linkage measures values. It allows to identify the key industries, as well the backward-linked and forward-linked industries in the economy.
The axes intercept in the point (1;1) and separate the key industries (with the value of BL and FL greater than 1) in the first quarter – these are:
- Industrial products (P),
- Machinery industry (M),
- Constructions (C),
- Trade (T),
- Transport (D).
In the second quarter there are important forward linkage industries:
- Mining and quarrying (Q),
- Professional, scientific and technical services (N).
The fourth quarter contains important backward linkage industries:
- Food industry (B),
- Financial, insurance and real estate services (F).
And the third quarter contains the rest of the sectors with lowest backward and forward linkages.

Similar figures for the rest of the countries are provided in Appendix 2.

Economic landscape of the Polish economy based on the Multiplier Product Matrix (before and after ordering the industries) is presented in figure 4 and 5.
Appendix 3 presents the economic landscape for the rest of the countries.

In order to compare different landscapes of the countries the Euclidean distances between landscapes are calculated and a tree diagram is constructed. Figure 6 presents
grouping the countries according to Euclidean distance between their economic landscapes. Countries with the similar structure of production form the clusters.

![Graph showing Euclidean distance between countries](image)

**Figure 6. Grouping the countries according to Euclidean distance**

The production structure of the Polish economy is similar to those of Slovenia, Italy and Finland.

**Conclusions**

Backward and forward linkages based on input output analysis allow to identify key industries in the economy, as well as important backward-linked and forward-linked sectors. In order to take into account the relative share of the industry in the economy, it is necessary to apply weights to the elements of the Leontief inverse matrix. The applied weights in the presented study are the obtained as the ratio of the industry’s intermediate transactions to the total intermediate transactions in the economy.

The comparative analysis of selected European countries revealed that the structure of production differs among the countries. The differences concern the identified key industries, the strength of linkages between industries and in effect the economic landscape of the country. It is worth to mention, that there are possible other aggregation rules for the industries depending on the purpose of the investigation.

The analysis shows that key industries of the Polish economy are: Industrial products, Machinery industry, Construction, Trade and Transport services. Forward linkage industries are: Mining and quarrying and Professional, scientific and technical services. Backward linkages industries are: Food industry and Financial, insurance and real estate services. Comparison to other countries’ economic landscape shows that the production structure of Polish economy is similar to that of Slovenia, Italy and Finland.

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Appendix 1.
Appendix 3.