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Verification of selected market microstructure hypotheses for a Warsaw Stock Exchange traded stock

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Abstract

This paper analyses the properties of the transaction process for the most liquid stock traded at the Warsaw Stock Exchange, namely Bioton (ISIN: PLBIOTN00029), in the light of market microstructure theory. The Autoregressive Conditional Duration and Autoregressive Conditional Multinomial models are estimated for the transaction process. Estimation results are interpreted in favour or against market microstructure hypotheses. Tests are conducted for the ACD models in order to assess their fit to the data and in order to search for ways of improving fit. The article is a follow-up of research by Bień [1].

1 Introduction

Market microstructure is the study of trade processes in goods or assets and particularly the study of price formation and behaviour of observable variables such as prices, price volatility, transaction volumes and transaction process intensity.

Growing interest in market microstructure theory and in analysis of transaction data is a phenomenon possibly due to advances in IT technologies applied to financial market operation. Over the course of years, researchers gained access to more and more frequently recorded data on financial instrument trading. On the one hand theoretical models of investor behaviour were developed in order to capture the types of incentives investors respond to, on the other ultra-high frequency data required developing new tools for data analysis which would capture characteristics such as unequal times between observation points, data discreteness etc.

This work is based on the author's master's thesis [2] and concentrates on inferences from information models whose development stems from the work by Grossman [3] and Copeland and Galai [4] and their implications for the properties of the transaction process. Time series data on the transaction process is analysed with the use of Autoregressive Conditional Duration and Autoregressive Conditional Multinomial models, introduced by, respectively, Engle and Russell [5] and Russell [6]. The research was conducted under the supervision of dr Katarzyna Bień and was an extension of her own research of the Polish stock market microstructure.

In section 2 a brief description of models by Diamond and Verrecchia [7] and by Easley and O'Hara [8] is given and their implications are summarised. Section 4 presents the properties of observed data. Section 3 briefly explains the ACD and the ACM models. In section 5 the models are put to use and results are presented. Finally, section 6 concludes.

2 Market microstructure

This work analyses a price-driven market for a particular security and employs two information-based models by Diamond and Verecchia [7] and by Easley and O'Hara [8] to build several hypotheses on what the properties of the transaction process should be.

At the time of gathering of the data the Warsaw Stock Exchange operated Monday through Friday from 9:30 AM to 4:30 PM. Prior to beginning of trading the investors played an iterative single-price auction game to set the first price of the day. Afterwards the process continued to 4:10 PM when trading was

halted for 10 minutes and the auction was repeated to set closing prices. At 4:20 PM trading was restarted but all trades had to be executed at the closing price.

Even though the trading process does not involve constant presence of market makers, it is assumed, as in other work on the subject, that the implications of information-based models remain true.

Diamond and Verrecchia [7] present a model which is an extension to the one of Glosten and Milgrom [9]. They assume that in the market there is a risk-neutral perfectly competitive market maker and there are traders who trade a single security whose value is stochastic. Some of the traders have access to private information (hence they are referred to as *informed traders*) and the rest can only access public information and are assumed to trade for liquidity preference reasons (hence they are referred to as *liquidity traders*). Moreover, the traders are further divided into groups, irrespective of their information access. These groups differ in possibilities of short selling: some face no constraints on short sales, some face high costs of short selling whereas the rest can not short the traded security at all. The traders engage in trades with the market maker sequentially and the only observable facts at each moment are:

- if there was trade at all;

- the price of the security and transaction type: a buy or a sell.

Private information consists of precise knowledge of the future liquidation value of the security whereas public information consists of beliefs about the distribution of the liquidation value and about the number of traders who belong to particular groups in the population. Private information is revealed to liquidity traders and the market maker through trades. Basing on public information, agents can condition the probability distribution of the security value on observed events. This way they are engaged in a learning process.

The most important implications of the Diamond and Verrecchia model are:

- time between transactions is an important factor for the price formation process;

- in markets with short-sales constraints information revelation is slower than under no constraints which leads to a wider bid-ask spread on the market maker's quotes;

- long periods between transactions imply that private information is 'bad news', i.e. it says that the liquidation value of the asset is 0;

- long periods between transactions later cause more abrupt price movements when bad news finally is revealed, increasing price volatility.

Easley and O'Hara [8] analyse a similar framework. In their model, however, all agents, informed and uninformed, are assumed to be able to sell the asset. Private information, as earlier, is equivalent to knowing the future liquidation value of the asset. Contrary to the Diamond and Verrecchia paper, the Easley and O'Hara model assumes that agents face uncertainty of the existence of an information signal.

If an information signal exists, the market maker faces informed traders and liquidity traders. If it does not, the market maker only faces liquidity traders. Public information consists of beliefs about the probability distributions of the asset value, the information signal existence and value and, finally, the relative number of (potentially) informed and uninformed traders in the population. Through observing events such as trades and no-trade events, agents update their beliefs on the value of the information signal throughout the trading process. Again, this is the process of private information revelation. This setting implies that:

- observing a no-trade event implies that there was no arrival of an information signal;

- long periods between trades imply that there is no new information which makes the market maker narrow the bid-ask spread;

- the existence of an information signal makes the informed traders more willing to trade, therefore high trading volumes are related to the existence of new information which is revealed through trades and price changes—it causes increases in volatility.

These implications from the two competing theoretical models built around the same framework will are verified in section 6.

3   The ACD and ACM models

Economic phenomena require appropriate tools to be employed for their analysis. Financial market researchers who analyse ultra-high frequency data found regularities in the following areas:

- autocorrelation patterns for rates of return of price series;

- seasonal patterns in intraday data;

- properties of rate of return distributions (see Dacorogna et al. [10]).

Strong negative autocorrelation at the 1st lag and little to none autocorrelation at subsequent lags is a common feature of ultra-high frequency data, both in calendar and transaction time. The pattern can be observed at measurement frequencies of up to a few minutes and it is present in most asset price data. Ahn et al. [11], however, point that this effect is not observed in data on the behaviour of stock indices and certain stocks. This patterns is usually attributed to the bid-ask bounce phenomenon, first described by Roll [12].

The effects of public information announcements also drew attention of academics. Dacorogna et al. [10] point that price volatility is affected by *effective news*, i.e. the difference between the announced figures and the market forecast. The greater this difference is, the more abrupt price adjustments follow and the more investor activity occurs directly after the announcement. The effects of effective news are, however, transitory. In foreign exchange markets they decay in under 15 minutes (Almeida et al. [13]) but in less liquid markets the effects may be more persistent – Franke and Hess [14] find them decay after 10 to 60 minutes in the German bund futures market.

Recent developments in research on rate of return on commodity prices and foreign exchange rates distributions points that as measurement frequency increases, the tails of the distribution become fatter (Dacorogna et al. [10]). Moreover, due to the fact that almost in every market there exists a minimum price change denoted the 'tick', at high frequencies price changes are discrete and measured in numbers of ticks. For most assets price changes under 3 ticks constitute more than 95% of all observed price changes (see e.g. Bień [1]). A model for accounting for price changes, due to Russell [6], is covered in section 3.2.

Finally, it should be noted that if data is recorded in transaction time, i.e. when each data point corresponds to a single transaction (unless there is more than one transaction in the same second, see section 4), then it is irregularly spaced. Times between transactions, i.e. intertrade durations, are stochastic and may contain information value (this is implied by many market microstructure models, [8], [7] among others). A model accounting for intertrade durations, due to Engle and Russell [5, 15], is presented in section 3.1.

3.1 The Autoregresive Conditional Duration model

The Autoregressive Conditional Duration model was introduced by Engle and Russell [5, 15]. It allows for modeling the conditional density of intertrade

durations. Let $\{t_i\}_{i=1}^{N}$ be a series of transaction moments and let $x_i = t_i - t_{i-1}$ be the time between transactions, i.e. the intertrade duration. Then the conditional expectation of x_i is:

$$E(x_i|x_{i-1}, \ldots, x_1) = \Psi_i(x_{i-1}, \ldots, x_1; \theta) \tag{1}$$

where θ is the parameter vector of the conditional mean equation. It is assumed that intertrade durations are random variables of the form

$$x_i = \Psi_i \epsilon_i, \tag{2}$$

and that the random variables ϵ_i are *iid* and such that:

$$E(\epsilon_i) = 1 \qquad Var(\epsilon_i) = \sigma^2, \tag{3}$$

and that the ϵ_i are distributed along the real positive line with 0. The conditions in (3) are necessary for model identification (see Bauwens and Giot [16] and Bień [1]).

Numerous specifications for the conditional mean equation were proposed. The linear (proposed by Engle and Russell [5, 15]) and the logarithmic (proposed by Bauwens and Giot [17]) were employed in this study:

$$\Psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \Psi_{i-j} \tag{4}$$

$$\log \Psi_i = \omega + \sum_{j=1}^{p} \alpha_j \log x_{i-j} + \sum_{j=1}^{q} \beta_j \log \Psi_{i-j}. \tag{5}$$

If specification (5) is employed, (2) changes to:

$$x_i = \exp(\log \Psi_i)\epsilon_i. \tag{6}$$

The processes generated by such specifications are stationary if $\sum_{j=1}^{p} \alpha_j + \sum_{j=1}^{q} \beta j < 1$ (for the linear model (4)) or if $\sum_{j=1}^{q} \beta j < 1$ (for the logarithmic model (5)). The linear specification also demands that all model parameters be greater than zero. This is the reason behind using the logarithmic specification in this study. Both specifications may be extended towards using exogenous explanatory variables.

There are many options for choosing the distribution for ϵ_i, as long as it is a probability distribution on the real positive line with zero. Engle and Russell [5, 15] used the exponential distribution and considered using the Weibull distribution. Bień [1] described the uses of the exponential, Weibull, generalised gamma and Burr distributions and used three out of four of them, leaving the

Weibull distribution out. Bauwens and Giot [16] employed all of the afore-mentioned distributions. In this study, the Weibull distribution is used for estimation as it is an extension to the exponential distribution.

If the Weibull distribution is used, the conditional density function for ϵ_i is:

$$f(\epsilon_i | I_{i-1}) = \frac{\gamma}{\epsilon_i} \left[\frac{\epsilon_i \Gamma\left(1 + \frac{1}{\gamma}\right)}{\Psi_i} \right]^\gamma \exp\left(- \left[\frac{\epsilon_i \Gamma\left(1 + \frac{1}{\gamma}\right)}{\Psi_i} \right]^\gamma \right), \qquad (7)$$

where $\Gamma(\cdot)$ denotes the gamma function. The Weibull density function in (7) was reparametrised with $\mu = \Gamma\left(1 + \frac{1}{\gamma}\right)$ to assure that the mean of ϵ_i is 1. If $\gamma = 1$ the Weibull distribution becomes the exponential distribution. One of the main reasons behind expanding beyond the exponential distribution is that the hazard function of a Weibull distribution is not constant, contrary to the exponential distribution. The hazard function value might be interpreted as the probability that a transaction might occur in an infinitesimal time period. An increasing hazard function, occurring if $\gamma > 1$, implies that the probability of observing a transaction increases, the more time passed since the former transaction. If the hazard function is decreasing (i.e. $\gamma < 1$), the probability that a transaction is observed decrease, the more time passed since the former transaction. A value of $\gamma < 1$ implies that transaction clustering is observed, i.e. that transactions occur in groups.

The models are estimated by maximum likelihood. If the assumption about the density of ϵ_i is correct, the estimators are unbiased, asymptotically efficient and asymptotically normal (see Verbeek [18]).

3.2 The Autoregressive Conditional Multinomial model

Transaction data on price changes require using specific tool which will account for data discreteness and for the bid-ask bounce. If the price change usually takes no more than a few values, i.e. 1 or 2 ticks, the Autoregressive Conditional Multinomial model might be a good tool for analysing the properties of the transaction process. The model was introduced by Russell [6] and Russell and Engle [19] and bears some resemblance to the ordered probit model of Hausman, Lo and MacKinlay [20]. The presentation of the model follows Russell and Engle [19].

Assume that the price change expressed in price ticks is a random variable Y_i that may take one of k states at time i. Then let X_i be a vector whose jth row is 1 if Y_i takes the j-th state and let Π_i denote a vector of conditional probabilities that the j-th element of X_i takes the value 1. The price change process then follows a Markov chain:

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$$\Pi_i = PX_i \qquad (8)$$

where P is a transposed Markov chain transition matrix of conforming size. It is assumed that based on the information set I_i, P changes over time which yields estimation difficulties.

Russell and Engle account for these problems through transforming the model. One state of the Y_i variable, say state n, is chosen to be the base state. Subsequently, logs of odds ratios of the variable Y_i taking the m-th state against taking the n-th state are written as:

$$\log\left(\frac{\pi_{im}}{\pi_{in}}\right) = \log\left(\sum_{j=1}^{k} P_{mj}X_{(i-1)j}\right) - \log\left(\sum_{j=1}^{k} P_{nj}X_{(i-1)j}\right) = \sum_{j=1}^{k}\log\left(\frac{P_{mj}}{P_{nj}}\right)X_{(i-1)j} + c_m,$$
$$(9)$$

where π_{ij} is the j-th element of Π_i and P_{ij} is the j-th element of the i-th row of the P matrix.

The $k-1$ elements $\frac{P_{mj}}{P_{nj}}$ are collected in a matrix P^*. The probabilities of the Y_i variable taking states $1,\ldots,k$ may be computed directly from the matrix. The probability of the variable Y_i taking state n follows from the restriction that the probabilities sum to 1. Moving from direct modeling of the transition matrix towards modeling logs of odds ratios allows for the following dynamic specification of the odds ratio equation:

$$\log\left(\frac{\pi_{im}}{\pi_{in}}\right) = \sum_{j=1}^{p} A_j(X_{i-j} - \Pi_{i-j}) + \sum_{j=1}^{q} C_j \log\left(\frac{\pi_{i-j,m}}{\pi_{i-j,n}}\right) + \gamma z_i, \qquad (10)$$

where z_i contains exogenous variables and the constant, Π_i is the vector of transition probabilities at time i and X_i is a vector whose jth row is 1 if the jth state of the Y_i variable occurred, except for the state n, if this occurs then X_i is a vector of zeros. Equation (10) allows for computing conditional probabilities of the variable Y_i taking the lth state:

$$\pi_{il} = \frac{\exp\left(\sum_{j=1}^{p} A_{jl}(X_{i-j} - \Pi_{i-j}) + \sum_{j=1}^{q} C_{jl} \log\left(\frac{\pi_{i-j,l}}{\pi_{i-j,n}}\right) + \gamma z_i\right)}{1 + \sum_{m=1}^{k-1} \exp\left(\sum_{j=1}^{p} A_{jm}(X_{i-j} - \Pi_{i-j}) + \sum_{j=1}^{q} C_{jm} \log\left(\frac{\pi_{i-j,m}}{\pi_{i-j,n}}\right) + \gamma z_i\right)}.$$
$$(11)$$

The model is easier to interpret if written in the ACM(1,1) specification. The matrix A then determines determines the impact of information from previous periods on current transition probabilities and the eigenvalues of the matrix C indicate how quickly this impact fades. If all transition probabilities are to be

non-zero, the condition that all solutions of the equation in r, $|I - C_1 r - C_2 r^2 - \ldots - C_q r^q| = 0$, are smaller than 1 in absolute value must be satisfied.

The model is estimated by maximum likelihood. Write $\frac{\pi_{im}}{\pi_{in}} = Z_i' \beta_m$. Then the log likelihood of the sample is:

$$\log L = \sum_{i=1}^{N} \left\{ \sum_{j=1}^{n-1} x_{ij} \left[Z_i' \beta_j - \log \left(1 + \sum_{k=1}^{n-1} \exp(Z_i' \beta_k) \right) \right] - x_{in} \log \left[1 + \exp \left(1 + \sum_{k=1}^{n-1} \exp(Z_i' \beta_k) \right) \right] \right\}$$

$$(12)$$

and estimation is carried out with the use of GAUSS.

4   Empirical properties of data

The data set consists of records on 32,345 transactions in common stock of Bioton S.A. (ISIN: PLBIOTN00029) that took place over the course of 15 trading days from Aug 1st 2007 to Aug 22nd 2007. The data was acquired through an evaluation version of the *Notowania 2* software developed by *STATICA* who is a data disseminator of transaction data from the Warsaw Stock Exchange. The data sheet is a chronological record of all transactions, as presented in table (1). Data on transactions recorded at the same time was aggregated into a single data point whose volume is total volume of the transactions at that time and whose price is the price of the last transaction at that time. The data in rows $4 - 7$ of table (1) probably represents a market sell order of 4,000 shares being executed against four buy limit orders. After aggregation and discarding transactions that took place prior to 10:00 AM and after 4:00 PM 11,605 data points were obtained. The early and late-time observations were discarded in order the clear the data of the effects of the orders submitted at the initial opening auction. This is common practice in market microstructure research, see e.g. Bień [1], Engle and Russell [5] and Bauwens and Giot [17].

4.1   Intertrade durations

The shares of Bioton S.A. are among the most often traded ones at the Warsaw Stock Exchange. Intertrade durations in seconds are presented in figure (1). It should be noted that in the period analysed almost half of the transactions recorded took place over the last 5 days of the sample. This means that between Aug 14th and Aug 22nd trading intensity was higher than in the former part of the sample. As seen in figure (1), in the earlier part of the sample intertrade durations are longer.

The period of August 2007 was when the current financial crisis was beginning to unfold saw many events of what in microstructure is called 'bad news'

Table 1: Database fragment – transactions on Bioton S.A. shares on Aug 3rd 2007

Date	Time	Transaction Price	Transaction Volume
Aug 3rd 2007	09:50:36	1.55 PLN	1170
Aug 3rd 2007	09:50:43	1.54 PLN	7000
Aug 3rd 2007	09:51:50	1.55 PLN	1213
Aug 3rd 2007	09:52:02	1.54 PLN	19
Aug 3rd 2007	09:52:02	1.54 PLN	2000
Aug 3rd 2007	09:52:02	1.54 PLN	1000
Aug 3rd 2007	09:52:02	1.54 PLN	981

arrivals. Over the sample period the price of Bioton S.A. shares fell 16.2% as seen in figure (2).
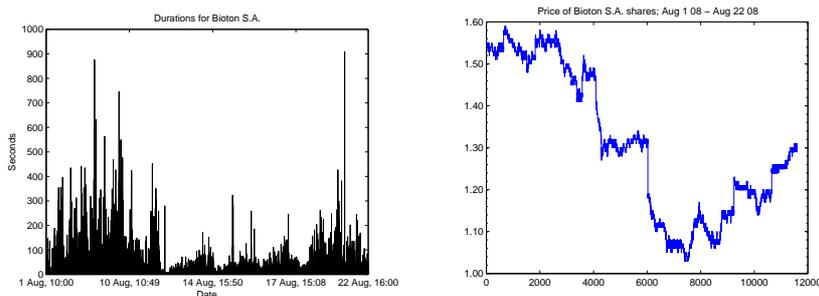
Figure 1: Intertrade durations for Bioton S.A. shares from Aug 1st 2007 to Aug 22nd 2007

Figure 2: Bioton S.A. share price from Aug 1st 2007 to Aug 22nd 2007; data source: WSE, own calculations

Sample statistics for intertrade durations are presented in table (2). The distribution of durations is skewed to the right as its mean is much higher than the median. A common characteristic, often observed in intertrade duration data (see, e.g. Engle and Russell [5, 15], Bauwens and Giot [17, 16] and Bień [1]) is overdispersion, i.e. the coefficient of variation being greater than 1.

Table 2 also presents evidence on the existence of autocorrelation in intertrade durations. Even though autocorrelation is not strong, even in short lags, it decays very slowly over time (see also figure (3)) and is highly statistically significant as indicated by Ljung-Box statistics for 10 and 100 lags reported in table (2). Respective critical values for the reported autocorrelation tests are 18.31 and 124.34.

Table 2: Descriptive statistics for intertrade durations on Bioton S.A. shares; data source: WSE, own calculations

No. of observations	11,604	Minimum	1 s
Mean	27.92 s	Maximum	904 s
St. Dev.	47.3 s	ACF(1)	0.2689
Median	12 s	LB(10)	6,335.68
Coefficient of Variation	1.694	LB(100)	38,735.4

As noted by many researchers, persistent autocorrelation might be caused by intraday seasonal factors coming into the picture. Out of many ways of removing intraday seasonality the FFF method, introduced by Gallant [21, 22] and applied by Andersen and Bollerslev [23] and by Bień, Nolte and Pohlmeier [24]. As intraday seasonality is assumed to be of multiplicative form, i.e. the duration x_i can be decomposed into a stochastic component $\overline{x}_i$ and a deterministic component s_{t_i}:

$$x_i = \overline{x}_i s_{t_i}. \tag{13}$$

In order to remove intraday seasonality it is sufficient to divide the duration x_i by estimated the estimated value of the seasonal factor s_{t_i}. Estimated seasonal factors are presented in figure (4). It can be observed that they assume the typical inverted-U shape, which implies that intertrade durations are longer in the middle of the trading day and shorter soon after the market opens and before the market closes. From this point on, deseasonalised intertrade durations are referred to as intertrade durations.
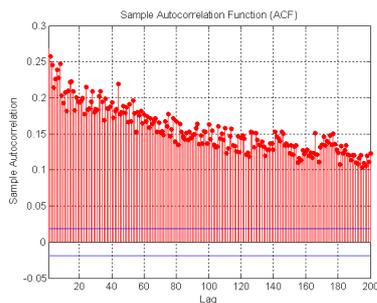

Figure 3: Autocorrelation function of intertrade durations on Bioton S.A. shares; data source: WSE, own calculations
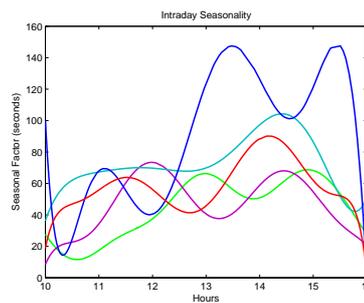
Figure 4: Intraday seasonal factors for intertrade durations on Bioton S.A. shares; data source: WSE, own calculations

After intraday seasonality is accounted for, autocorrelation in intertrade du-

rations decays exponentially and it's statistically insignificant after the 5th lag as shown in figure (5). The descriptive statistics for deseasonalised data are reported in table (3). The Ljung-Box statistics decreased by almost 2 orders of magnitude, but they still imply that there exists autocorrelation in intertrade durations. The properties of the duration process are now, however, desirable: the ACF decays exponentially and the mean is close to 1. As the coefficient of variation is 1.412, the data still exhibit overdispersion. Moreover, squared intertrade durations exhibit no autocorrelation (see figure (6)): the Ljung-Box statistics for 10 and 100 lags are, respectively, 1.41 and 4.068.

Table 3: Descriptive statistics for deseasonalised intertrade durations on Bioton S.A. shares; data source: WSE, own calculations

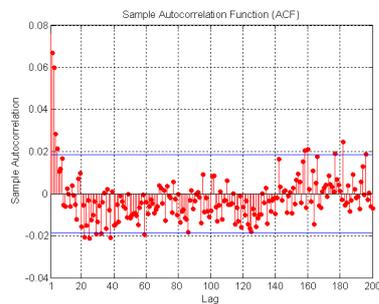| No. of observations	11,604	Minimum	0.0069
Mean	1.0101	Maximum	72.3421
St. Dev.	1.426	ACF(1)	0.0761
Median	0.6147	LB(10)	182.9
Coefficient of Variation	1.412	LB(100)	278.2

Figure 5: Autocorrelation function for deseasonalised intertrade durations on Bioton S.A. shares; data source: WSE, own calculations
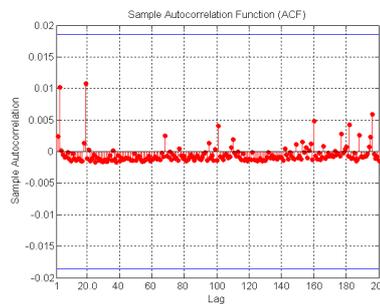
Figure 6: Autocorrelation function for squared deseasonalised intertrade durations on Bioton S.A. shares; data source: WSE, own calculations

Nonparametric analysis of the unconditional distribution of logarithms of deseasonalised intertrade durations and of their empirical hazard function suggests that the exponential distribution is not appropriate for the process. The kernel estimate of density crosses the density function of the Gumbel distribution twice which means that the empirical CDF would cross the Gumbel distribution CDF once. (The natural logarithm of an exponentially distributed variable

has a Gumbel distribution). This fact implies that there is higher probability of assuming very large values by the logs of intertrade durations than if these were Gumbel-distributed. Moreover, the empirical hazard function increases in $log(x_i)$ more slowly than the hazard function of the Gumbel distribution. This implies that the hazard function of intertrade durations x_i should be downward sloping, whereas the hazard function of the exponential distribution is constant. These findings suggest that the Weibull distribution might be more apt at capturing data characteristics. It is expected that the estimate of the γ parameter is below 1 (see section 3.1).

4.2   Transaction price changes

At ultra high frequencies price changes are discrete, i.e. the prices move in ticks and the moves are usually small. For Bioton shares in 68.17% of transactions no price changes were observed and the price moved 1 tick upwards or 1 tick downwards in 15.76% and 15.38% of transactions respectively. Greater changes occurred in only 0.69% of transactions. The price tick size for Bioton shares is 0.01 PLN.

Contrary to findings in data on intertrade durations, the data on price changes seems not to be exhibiting any major changes across the sample. Looking at data in greater detail shows that price changes exhibit clustering (see figure 7), i.e. after a transaction with a price change was observed, the probability of observing subsequent price changes increases. Moreover, price changes exhibit strong negative autocorrelation of order 1.

Two dummy variables were constructed from the price change data, following propositions by Bień [1]: d^+ and d^-. These variables are indicators of price changes occurring. The value of d^+ is 1 in periods when the price increases and 0 otherwise, the value of d^- is 1 in periods when the price decreases and 0 otherwise. Apparently the price change process is not symmetric, i.e. price increases are followed by price decreases more often than price decreases are followed by price increases. The plot of the cross correlation function of variables d^+ and d^- is shown in figure 8. The figure should be interpreted in the following way: the correlation of the current value of d^+ and the lagged value of d^i is shown at positive lags, whereas the correlation between current values of d^- and past values of d^+ is shown at negative lags. The cross-correlation function is almost symmetric with one important exception: current values of d^- exhibit stronger correlation with first-lag values of d^+ than current values of d^+ with first-lag values of d^-. This observation is an important implication for the specification of the ACM model.

Figure 7: Price changes of Bioton S.A. shares; data source: WSE, own calculations
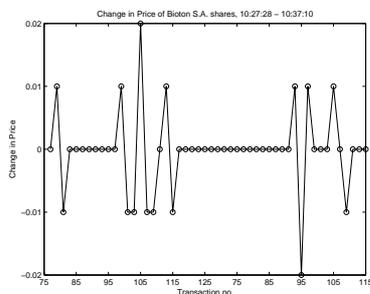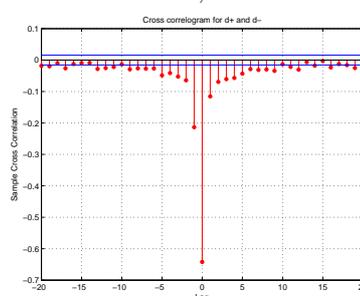

Figure 8: Cross-correlation function of d^+ and d^- auxiliary variables; data source: WSE, own calculations

5 Properties of the transaction process

In this section the properties of the transaction process will be presented. That is, estimation results of best-fitting ACD and ACM models will be presented and commented upon.

5.1 Specification of the ACD model

The best-performing among the estimated ACD models use the logarithmic specification due to Bauwens and Giot [25] and the Weibull distribution for innovations. The former is motivated by the fact that the kernel density estimate of the hazard function of logs of unconditional durations is increasing, yet lower than the theoretical hazard function of the logarithm of an exponentially distributed variable (which is an exponential function). The latter is motivated by the fact that the logarithmic specification allows for more flexibility in terms of e.g. extending the set of explanatory variables. Moreover, following Bauwens and Giot [25] and Bień [1], the ACD models were augmented with the following explanatory variables:

- v_{i-k}, $k = 0, \ldots, 3$, for transaction volumes; this variable is introduced in order to determine whether transaction volumes convey information;

- dp_{i-k}, $k = 1, 2, 3$, for transaction price changes, lagged by 1 and 2 observations; the introduction of this variable allows to determine if transaction intensity changes after good news or bad news arrives;

- px_i, the product of a binary variable assuming the value 1 if the price changed at observation i and the time since the last price change; this variable is believed to be a measure of price volatility

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- xv_i, the product of a binary variable assuming the value 1 if cumulative volume since the last occurrence of such an event is greater than or equal to 15.000 and the time since such an event occurred; this variable allows for measuring volume durations.

5.2   Specification of the ACM model

Price changes are sparse in the data and price changes greater than one price tick only appear in 0.69% of transactions. This means that a parsimonious three-state ACM model can be adopted for analysing the price change process. The state variable Y_i assumes the value of 1 if the price increases in transaction i, -1 if the price decreases and 0 if the price does not change. No price change is chosen as the base case. This means that the ACM model consists of two equations and that the explanatory variables are: the log of the odds ratio of $Y = -1$ vs. $Y = 0$ and the log of the odds ratio of $Y = 1$ vs. $Y = 0$.

Cross correlations of the d_i^+ and d_i^+ variables imply that the only restrictions that should be applied to the model are those of Russell and Engle, i.e. that the parameter matrices are symmetric (see [19])

Due to the fact that ACM models tend to have many parameters, only $ACM(1,1)$-type models are estimated. The basic model, denoted $ACM(1,1)_0$ contains only the standard set of explanatory variables as mentioned in section 3.2. The augmented model, denoted $ACM(1,1)_1$ contains the following explanatory variables:

- v_i, the volume of transaction i;

- x_i, the duration for transaction i.

5.3   Estimation results - ACD models

The 18 competing models that were estimated were evaluated on the basis of likelihood ratio tests. First, the estimation results of the best-performing model is reported and commented upon. Subsequently, tests of several statistical hypotheses are reported and commented upon. Complete estimation results are available from the author at request.

Estimation was carried out with the use of MATLAB 7.1. The estimation code was developed by Marcelo Perlin from Reading University's ICMA and then adapted for estimating log-ACD models with additional explanatory variables by the author. The reported t-statistics were computed with the use of the OPG estimator. Statistical significance of variables was assessed on the basis of the

t-test results. Model quality was assessed with the use of the density forecasts technique (as introduced in Diebold, Gunther and Tsay [26] and later on used by Bauwens, Giot, Grammig and Veredas [27], and by Bień [1]) and through the Bayesian information criterion.

Table 4: ACD model estimation results; data source: WSE, own calculations

Variable	Parameter estimate	Variable	Parameter estimate
		ACD(3,3) model	
ω	-0.4571 (0.1530)	dp_{i-2}	2.6635 (0.1756)
x_{i-1}	0.07917 (11.6650)	dp_{i-3}	2.6011 (0.3202)
x_{i-2}	0.1192 (9.8972)	v_i	-0.02901 (-4.1013)
x_{i-3}	0.08083 (3.4132)	v_{i-1}	-0.04813 (-5.1655)
Ψ_{i-1}	-0.84988 (-4.6185)	v_{i-2}	-0.03380 (-4.5994)
Ψ_{i-2}	0.0130 (3.2554)	v_{i-3}	-0.0040 (-1.1461)
Ψ_{i-3}	0.6561 (5.3411)	px_i	3.3878e-005 (1.2877)
dp_{i-1}	2.6635 (0.1756)	vx_i	1.1087e-005 (0.3340)
γ		0.9484 (21.4336)	
		Other	
log-likelihood		-11491.804	
BIC		22824.507	
DFcst		409.36	

From among the extra explanatory variables the resulting model exhibits statistically significant estimates of parameters at v_i and its lags. Even though the remaining extra explanatory variables are not statistically significant, the model performs better than any other estimated model in terms of the BIC and the likelihood ratio test. It is also worth to notice that the γ parameter estimate is $0.9484 < 1$ which implies that the probability of a new trade occurring decreases as the intertrade duration increases. These and other estimation results are commented upon in section 6.

The intertrade duration process exhibits rich dynamics which is well-captured by the (3,3) model. Residual autocorrelation decreases and formal tests do not allow to reject the hypothesis of no remaining autocorrelation. The test is conducted with the use of the procedure introduced by Meitz and Teräsvirta [28]. For that purpose consistent parameter estimates are required. The reported estimates can not be used for this procedure as the hypothesis that the ϵ_i's are Weibull distributed is rejected by the density forecast test. Hence, the estimates are not consistent. However, if the model is estimated by QML, under

the assumption that the ϵ_i's are exponentially distributed, the estimates are consistent, yet possibly biased. Such a model was estimated, the reported LM statistic of 2.305 vs. a $\chi^2(5)$ critical value of 11.07. This result confirms that a log-ACD(3,3) model correctly captures the dynamic structure of intertrade durations.

The aforementioned density forecast technique allows for testing whether the assumptions on the distribution of the innovations ϵ_i are correct. The technique requires, first, that for each x_i its conditional CDF is computed (these are further denoted as z_i's). If the distributional assumption is correct, then the z_i's are independently and uniformly distributed on $(0,1)$ which is tested by the Pearson χ^2 test. The histogram of z_i values is presented in figure 9. The adequacy of the chosen innovation distribution is also evaluated with the use of quantile graphs in figure 11. The density forecast test statistic of 409.36 is significantly higher than its $\chi^2(24)$ critical value of 36.415. The true distribution of ϵ_i's is not Weibull. The test results, however, improve slightly when moving from exponentially to Weibull-distributed innovations. Improvement might also be seen in the quantile graphs presented in figures 10 and 11. Even though the Weibull distribution is less restrictive than the exponential distribution, it does not seem to be sufficient to capture all the distributional properties of the data. Better results are reported by many researchers for the generalised-gamma and the Burr distributions.

The intertrade duration process exhibits little persistence, i.e. the parameter values at Ψ_{i-1} are in general smaller than 0.70. The transaction process of a stock as liquid as Bioton S.A. seems to be driven by quick trades, each of which reveals very little information and has little impact on the process. In the reported model the parameter value is below zero which is due to the inclusion of many lags in the model. The parameter values reported in table 4 are further commented upon in section 6.

5.4   Estimation results - ACM models

As mentioned, two ACM models were estimated: a plain model and one with extra explanatory variables. The models were estimated in *GAUSS 3.2.37* with the use of the *maxlik* library and the code by prof. Roman Liesenfed which was made accessible by Katarzyna Bień, PhD. The estimation process, however, posed many problems related to sample size. The two models were estimated on samples of reduced sizes. The basic ACM model saw 7,000 observations whereas the augmented model saw 1,500 observations. The estimates of the parameters of the latter model are reported in table 5.
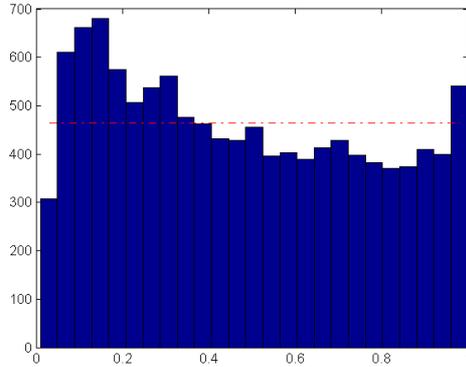
Figure 9: Histogram of density forecasts of the $ACD(3,3)_4$ model; data source: WSE, own calculations

Table 5: ACM model estimation results; data source: WSE, own calculations

Parameter	Estimate	Parameter	Estimate
c_{11}	0.5767 (11.143)	$\gamma_{1,c}$	0.5090 (1.875)
c_{12}	0.4186 (13.990)	$\gamma_{1,v}$	-0.0675 (-2.116)
c_{22}	0.2606 (3.536)	$\gamma_{1,x}$	0.0567 (1.608)
a_{11}	-0.3747 (-4.306)	$\gamma_{2,c}$	-0.0953 (-0.314)
a_{12}	0.8416 (15.882)	$\gamma_{2,v}$	-0.0804 (2.350)
a_{22}	-0.8225 (-3.785)	$\gamma_{2,x}$	0.0768 (1.633)

In both models the C parameter matrix has the important property that all of its eigenvalues are less than one in absolute value. This means that the models are correctly specified in terms of the properties of the price change process, as the estimated process should assume every possible states infinitely many times in the future. Moreover, the parameter values in the C matrix imply that the price change process is of little persistence, i.e. if some event increased the probability of a price change at time i then at time $i + 1$ the effect of this event will be significantly lower. The difference between estimates of c_{11} and c_{22} is statistically significant which might imply that investors discover bad news quicker than they discover good news, or that bad news has a greater impact at market participants' beliefs about the fundamental value of the stock.
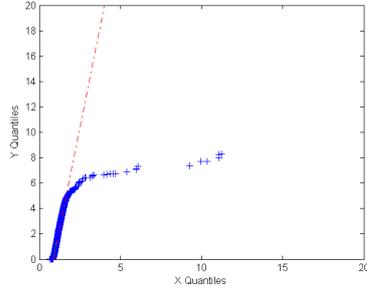
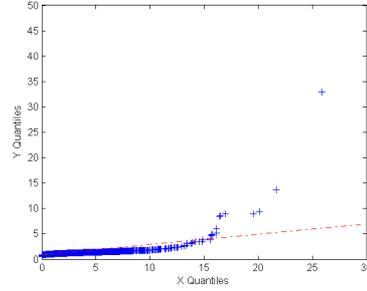Figure 10: Quantile plot of exponential-ACD model residuals; data source: WSE, own calculations

Figure 11: Quantile plot of Weibull-ACD model residuals; data source: WSE, own calculations

The estimates of the parameters on the diagonal of A, i.e. a_{11} and a_{22} are both negative, which means that after each price increase (decrease) the probability of another price increase (decrease) falls. The positive value of a_{12} implies that after the price of the stock changes, the probability of an opposite change increases; high value of a_{12} might be related to the importance of the bid-ask bounce observed in the data. It is an important fact that a_{11} is significantly higher than a_{22}. This result shows that the probability of consecutive price decreases falls slower than the probability of consecutive price increases.

The parameter estimates at the exogenous explanatory variables are all statistically significant at the 0.1 significance level (this level is assumed due to reduced sample size). Positive values of $\gamma_{1,x}$ and $\gamma_{2,x}$ imply that the probability of a trade occurring grows as the time since the last transaction increases. This result is in disaccord with the estimate of the γ parameter in the prevailing ACD model, which is smaller than 1, implying that the probability of a trade occurring decreases in time from last transaction. The values of parameters at $\gamma_{1,v}$ and $\gamma_{2,v}$ are both negative. This results does not agree with theoretical presumption nor with estimates of ACD models which imply that trading intensity increases in volume.

The ACM model removes residual autocorrelation quite well and the multi-dimensional autocorrelation test statistics are lower than their respective critical values for lags greater than 1. The autocorrelation and cross-correlation plots of the residuals are reported in figures 12 and 13.
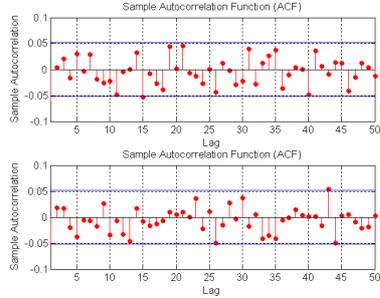
19

Figure 12: Autocorrelation func-
tion of ACM model residuals; data
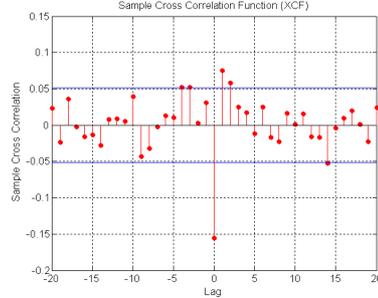source: WSE, own calculations

Figure 13: Cross-correlation func-
tion of ACM model residuals; data
source: WSE, own calculations

6 Estimation results and market microstructure hypotheses

This section summarises the findings and confronts them with the implications
of the theoretical models which were briefly described in section 2. First, the
hypotheses concerning parameter signs in both the ACD and ACM models are
summarised. Afterwards, the estimation results are commented upon in the
light of the aforementioned hypotheses.

In the ACD model it is expected that:

- v_i:

  Easley, O'Hara: An increase in v_i means that private information arrives
  which implies that trading intensity should increase; the parameter should
  be of negative sign.

  Diamond, Verrecchia: Increase in v_i means that good news arrived which
  implies that trading intensity should increase; the parameter should be of
  negative sign.

- dp_i

  Easley, O'Hara: Trading intensity does not depend on news being good
  or bad, the estimate should be zero.

  Diamond, Verrecchia: An increase in dp_i, interpreted as an arrival of
  good news, should result in an increase of trading intensity; the parameter
  estimate should be of positive sign.

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- px_i:

  Easley, O'Hara: Increases in px_i interpreted as increases in price volatility should result in increases in trading intensity; the parameter estimate should be of positive sign.

  Diamond, Verrecchia: Increases in volatility are interpreted as effects of the arrival of bad news and should result in a decrease of trading intensity; the parameter estimate should be of negative sign.

- vx_i:

  Easley, O'Hara: Greater volume of trades implies that some investors have private information which results in higher trading intensity; the parameter estimate should be of positive sign.

  Diamond, Verrecchia: Greater volume of trades implies that good news arrived to the market and trading intensity should increase; the parameter estimate should be of positive sign.

In the ACM model it is expected that:

- v_i:

  Easley, O'Hara: An increase in v_i means that private information arrives and that price volatility should increase; the parameter estimates should be of positive sign and equal in both equations.

  Diamond, Verrecchia: An increase in v_i implies that good news arrived to the market which should result in an increase of the probability of price increases; the parameter should be of positive sign in the $log(P(Y_i = 1)/P(Y_i = 0))$ equation and zero in the $log(P(Y_i = -1)/P(Y_i = 0))$ equation.

- x_i:

  Easley, O'Hara: An increase in x_i means that no new private information arrives which should result in decreasing price volatility; the parameter estimates should be of negative sign in both equations.

  Diamond, Verrecchia: An increase in x_i is interpreted as a result of the arrival of bad news which leads to an increase in price volatility; the parameter estimates should be of positive signs in both equations and greater in the $log(P(Y_i = -1)/P(Y_i = 0))$ equation than in the $log(P(Y_i = 1)/P(Y_i = 0))$ equation.

The estimation results of the ACD model allow for drawing the following conclusions:

- The parameter estimate at v_i is negative and statistically significant which is in accord with both theoretical models' predictions. Bień's [1] results indicated similar results for other WSE-traded stocks.

- The parameter estimate at dp_i is not statstically significant. As Easley, O'Hara [8] implies, the price change direction does not affect trading intensity. Bień's results indicated such behaviour for 5 out of 8 researched stocks. In case of the 3 remaining stocks the parameter estimates were positive.

- The parameter estimates at px_i and vx_i are not statistically significant. This result is in disaccord with both theoretical models analysed. It is also possible that the variable px_i is not a good measure of instantaneous price volatility and that the variable vx_i is not a good measure of instantaneous trading intensity.

Estimation results do not allow to confirm that the price formation process of Bioton S.A. stocks strictly follows any of the two theoretical models introduced by Easley and O'Hara or Diamond and Verrecchia. These conclusions should, however, be looked at critically as the chosen type of the ACD model did not fully capture the properties of the transaction process. ACD models require a more general innovation distribution than hereby used Weibull distribution which assumes a constant coefficient of variation. A three-lag model is, on the other hand, sufficient to capture the dynamic properties of the duration process, removing most residual autocorrelation. Under these reservations, it seems that in the case of Bioton S.A. stocks the Easley and O'Hara model describes the transaction process better than the Diamond and Verrecchia model. These findings are different from the results of research on the Polish stock market microstructure, conducted by Bień [1], who states that for the researched stocks the former model is fully rejected whereas the implications of the latter are confirmed to some degree.

The estimation of the ACM models allow for drawing conclusions which are not completely in accord with the implications of the estimation results of the ACD models:

- The parameter estimate at x_i is of positive sign in both equations. This implies that the more time passed from the last transaction, the greater is the probability that a trade will occur and that the price will change. This result is contradictory with the estimate of the Weibull distribution γ parameter being less than 1 in all ACD models. A $\gamma < 1$ in the Weibull

distribution implies that the probability of a trade occurring decreases as time from the last transaction increases.

- The estimates of the parameters on the diagonals of the A and C matrices (see section 5.4) are different across equations in both the basic and the extended model; the difference being greater in the latter. These findings imply that after a price change the probability of consecutive price drops decreases slower than the probability of consecutive price increases.

- The parameter estimate at $log(v_i)$ in the extended model is negative in both equations. This implies that the probability of a price change decreases in transaction volume. This result contradicts the estimation results of the ACD models and is in disaccord with both theoretical frameworks.

These results reject both theoretical models that were the sources of tested hypotheses. The most important implications are that the probability of a price change increases in duration (i.e. time that passed from the last transaction) and that the probability of a price change decreases in transaction volume. The latter results is similar to an observation by Bień [1] who finds it in contradiction with the results of Liesenfeld and Pohlmeier [29] and of Liesenfeld, Nolte and Pohlmeier [30].

7 Summary

To conclude, it is stated that the ACD and ACM models are useful tools of describing the transaction processes at the Warsaw Stock Exchange. Moreover, these models allow for testing the hypotheses of theoretical market microstructure models. The analysed transaction process, i.e. of Bioton S.A. stocks, has similar properties to those of Agora S.A., BPHPBK, BRE and BZWBK, Comarch, Kęty, PGF and TPSA which were described by Bień. Moreover, it can be stated that in the transaction process of a very liquid stock, such as Bioton S.A., very little amounts of information are revealed through each trade. In the price change process the probabilities of price changes react only marginally to changes in exogenous variables. The research also shows that the stock price does not reflect all information available to investors who should observe other market characteristics, such as trading volume and trading intensity, as well.

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