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Price-wage system with taxation: multivariate cointegration analysis

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Price-Wage System with Taxation: Multivariate Cointegration Analysis

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Summary

The paper investigates the price system and the wage equation in the presence of taxes. Price formation is analysed at three levels: producer's prices, trade in consumer goods and, separately, in services, and at the aggregate level of the cost of living index. This is in the spirit of classical macromodels that usually apply the "bottom-to-top" approach. However, because of nonstationarity of variables, this study employs multivariate cointegration. The empirical investigation is based on Polish monthly data covering the period from January 1993 to December 2003. Its results allow to conclude that as many as five stable long-run relationships drove inflation in Poland in that period. Appropriate decomposition of price formation made it possible to incorporate all conditions postulated by economic theory (i.e. homogeneity, unit elasticities) and to show how direct and indirect taxes impact decisions made by the employers and employees.

JEL classification: C32; E24; E31

1. Introduction

The prevailing paradigm for modeling national-economies in Europe is that firms have mark-up opportunities in the environment of imperfectly competitive goods and labor markets. Wages are determined through bargaining and real activity is independent of the (steady-state) inflation rate (see Wallis (2004)). In addition, the model is expected to show a non-accelerating inflation rate of unemployment, NAIRU, which implies static homogeneity of the price and wage equations. These statements about the long-run behavior are in fact testable hypotheses that can be verified in a careful modeling process. The major difficulty arises, however, from the nonstationarity of macroeconomic time series, which calls for pertinent methods, namely the multivariate cointegration analysis. It should be stressed that only the decomposition of variables' behavior into the long-run and short-run dynamics within the framework of a structural vector equilibrium correction model (SVEqCM) allows to include and test assumptions induced by economic theory.

The successful application of the SVEqCM is not straightforward, as the time series is rather limited in most cases. The strategy of reducing the parameter space and the size of the model itself turns out to be of crucial importance. The first can be achieved by marginalizing the model at an early stage of the analysis (originally proposed by Greenslade et al. (2000)). The other is a function of the number of variables included in an analysis. Therefore, it is a common practice to operate on a high level of aggregation and to test cautiously for (weak) exogeneity, as well as to check the properties of residuals. Should they prove to be Gaussian white noise, then the hypothesis of model completeness is supported.

The aim of the presented study was to build a model explaining the price-wage behavior in the presence of taxes and capable of capturing the main features of the economic system and potentially useful in policy simulation studies. Taking into account earlier results (see Welfe (2000), Welfe, Majsterek (2002), Welfe at al. (2004)) we are convinced that the model should distinguish between prices of goods and prices of services; additionally, influence exerted by the prices of imported consumer goods should be separated from the impact of prices of imported materials and intermediate products, as these two show significant differences in dynamics. It is also important to introduce direct and indirect taxes to the model, because both of them affect decisions that lead to the formation of wages and prices. To quantify these relationships properly, the analyzed model explains prices starting with the producer level, then proceeds to the consumer's level, to finally arrive at the price of living index. This approach is in the spirit of classical macromodels, which have usually been built "bottom-to-top", that is with components summed up to combine the aggregates (see Klein at al. (1999)). The size of the model is an obvious compromise between the needs and limitations arising from the estimation and testing procedures.

The paper is organized as follows. The main economic assumptions about the long-run behavior followed by model's structure are presented in section two. Section three explores data and properties of the time series used in the study. The SVEqCM model and inference on the lag length and the size of cointegrationg space conditional on weak exogeneity are discussed in section four. Estimates of the long-run relationships and the complete structure of the model with an economic interpretation of the empirical results are provided in section five. The paper ends with conclusions.

2. The long-run model's structure

Let us start by assuming that the producer prices are determined as a mark-up on imports and unit labor costs:

$$pr = \delta_1^r pm^I + (1-\delta_1^r)(w-z) \tag{1a}$$

where pm^I is the price of imported materials and intermediate products, w - nominal wages, and z – labor productivity. Small letters denote (natural) logarithms. The homogeneity assumption implies that coefficients add up to unity. From this equation it follows that wages can only affect inflation, when their increase is not compensated for by productivity growth. Theoretical foundations can be traced back to the cost-push inflation hypothesis (see classic work of Tobin (1972)).

It is commonly accepted that real wages depend on both productivity, z, and the labor market pressures measured by the rate of unemployment, U:

$$w - p = \delta_1^w z - \delta_2^w U\ . \tag{1b}$$

It is possible to include in this equation some extra variables capturing wage pressures that originate from various sources. Static homogeneity of wages implies that $\delta_1^w = 1$. This function results from the accepted bargaining model of wages and prices (see Nickel (1984), Layard et al. (1991)) and it can be regarded as a standard wage function as well (see Tobin (1995)).

In the simplest case the cost of living index p is assumed the same as the producer price index, pr, then the above two equations form the most aggregated model of price-wage behavior. In the real world, however, these two indices are different for four fundamental reasons. Firstly, the import of consumer goods changes the dynamics of

p independently of pr. Secondly, a value added tax is charged on domestic and imported goods and its rate may vary in time. Therefore, the deflator of consumer goods (commodities), pc, should be expressed as a weighted sum of the deflator of imported final goods, pm^F, and the producer price index, pr, enlarged by a value added tax, tv:

$$pc = (1-\delta_1^c)pm^F + \delta_1^c pr + tv. \qquad (2)$$

Thirdly, the cost of living index, p, is the weighted sum of prices of commodities and services, ps:

$$p = \delta_1^p pc + (1-\delta_1^p)ps. \qquad (3)$$

Fourthly, taxes are imposed on wages and this significantly modifies objectives driving the two parties involved in the wage-bargaining process. Because of the taxes, employers are concerned with the total real wage costs:

$$wc - p = w + tw - p \qquad (4a)$$

where tw represents employment taxes. Employees, however, are sensitive to the real net wage:

$$w - td - p = wc - tx - p \qquad (4b)$$

where td stands for direct taxes and tx denotes total overheads ($tx = tw + td$).

All in all, we considered a system composed of equations explaining:

- producer price

$$pr = \delta_1^r pm^I + (1-\delta_1^r)(wc - z), \qquad (5a)$$

- the deflator of consumer goods (commodities)

$$pc = (1-\delta_1^c)pm^F + \delta_1^c pr + tv, \qquad (5b)$$

- the deflator of services

$$ps = \delta_1^s pc + \delta_2^s h \,, \tag{5c}$$

- the consumer price index

$$p = \delta_1^p pc + (1-\delta_1^p) ps \,, \tag{5d}$$

- wages

$$wc - tx - p = z - \delta_2^w U \tag{5e}$$

where h is the share of services in total consumption and, when growing, it represents the potential market pressures.

Consequently, the consumer price index can be written as:

$$p = [\delta_1^p (1-\delta_1^s) + \delta_1^s](pr + tv + tm) + (1-\delta_1^p)\delta_2^s h \tag{6}$$

where $tm = (1-\delta_1^c)(pm^F - pr)$ and it can be interpreted as a 'tax' imposed due to high import prices (see also Wallis (2004)). In the long run the dynamics of services' prices does not differ from the dynamics of commodities' prices, $\delta_1^s = 1$, therefore everything that matters beside producer prices is the weighted share of services in total consumption and taxes:

$$p = (pr + tv + tm) + (1-\delta_1^p)\delta_2^s h \,. \tag{7}$$

According to model (5a)-(5e) the exchange rate accelerates inflation through the price index of imported materials and intermediate products (used in production), pm_t^I, and through the price index of imported consumer goods, pm_t^F, since both of them are by definition the product of the exchange rate and relevant world price indices. However, pm_t^I and pm_t^F, and consequently the exchange rate, are treated exogenously. Whether this simplification is acceptable is an empirical question, which can be answered by proper testing.

3. The data

The empirical investigation is based on Polish monthly, seasonaly unadjusted data covering the period from January 1993 to December 2003. Raw data comes from official publications by the Polish Statistical Office (GUS). Some common procedures had to be applied to get comparable data forming time series (detailed description in Welfe at al. (2002)).

The time between years 1993 and 2003 was the period when the Polish economy evolved from a centrally planned economy towards a market economy following the change of the political system. However, the first stage of the transition (i.e. years 1990-1992), when the draconian devaluation of Polish zloty and adjustment of prices took place, making the cost of living index go up over eightfold, was intentionally omitted (a wider discussion of historical developments in Welfe (2002)). A worth noting fact is the commonly accepted view that the Polish economy started to be demand driven only from 1993, in contrast to its supply-constrained nature in the past. Consequently, assumptions typical of the market economies can be applied. An additional, nevertheless important reason for skipping the first transition years was unavailability of the monthly data. The series are plotted in Figure 1.

<Figure 1. around here>

The sample covers 11 years and consists of 132 monthly observations. Whether this suffices to determine stable, long run relationships depends on the size of the model and variables variation. If the latter is large enough, the relatively limited samples provide an adequate amount of information about the long run. Due to the economic system transformation, Polish data shows extensive variability and thus promises substuntial empirical findings.

This study concentrates on the price-wage system in the presence of taxes. The model is therefore built around five relationships (5a) to (5e). The set of endogenous variables includes the price index of production sold (producer prices), pr_t, the price index of consumer goods, pc_t, the price index of services, ps_t, the consumer price index (cost of living index), p_t, total wage costs in industry (current prices), wc_t.

The set of potentially weakly exogenous variables consists of the price index of imported materials and intermediate products (used in production), pm_t^I, the price index of imported consumer goods, pm_t^F, productivity in industry measured by value added per worker (costant prices), z_t, unemployment rate, U_t, value added tax, tv_t, taxes imposed on wages, tx_t, and the share of services in total consumption (constant prices), h_t. Taxes are expressed as proportion and in fact they represent amounts contributed to the state budget.

It is a well-documented phenomenon that most macroeconomic variables are first difference stationary, despite some pieces of evidence that price indices can be second difference stationary (see Juselius (1999)). Results presented in Table 1 confirm that all variables are I(1)**,** even though the joint ADF-KPSS test (for small sample critical values see Kębłowski, Welfe (2004)) suggests that some of them could be considered I(2). For every variable at a minimum level (varying from 3 to 12) a number of lags were chosen in the ADF to remove serial correlation and to ensure normal distribution of residuals, which was verified by the cumulated periodogram test (see Durbin (1969)).

<Table 1. around here>

4. Structural VEqCM. Model marginalization

Since all variables proved to be I(1), the natural tool in the empirical analysis is the structural vector equilibrium correction model, SVEqCM (see, for example, Johansen (1988), Hendry (1995) and Lütkepohl (2004)):

$$\Delta \mathbf{y}_t \mathbf{A}_0 = \tilde{\mathbf{y}}_{t-1}\tilde{\mathbf{\Pi}} + \sum_{s=1}^{S-1} \Delta \mathbf{y}_{t-s}\mathbf{A}_s + \mathbf{d}_t^z \tilde{\mathbf{C}} + \boldsymbol{\varepsilon}_t \tag{8}$$

where:

$\tilde{\mathbf{\Pi}} = \mathbf{\Pi}\mathbf{A}_0$, $\mathbf{A}_s = \mathbf{\Gamma}_s \mathbf{A}_0$, $\tilde{\mathbf{C}} = \mathbf{C}\mathbf{A}_0$, $\boldsymbol{\varepsilon}_t = \boldsymbol{\xi}_t \mathbf{A}_0$, and

$\mathbf{A}_0$ - contemporaneous coefficients matrix,

$\tilde{\mathbf{y}}_t = [\mathbf{y}_t \vdots \mathbf{d}_t^x]$,

$\mathbf{y}_t = [y_{1t} \ldots y_{Mt}]$ - vector of M stochastic variables,

$\mathbf{d}_t^x = [d_{1t} \ldots d_{Nt}]$ - vector of N deterministic variables,

$\mathbf{d}_t^z = [d_{1t} \ldots d_{Pt}]$ - vector of P deterministic variables outside the cointegration relation,

$\mathbf{\Pi}$ - total impact multipliers matrix, which can be decomposed as $\mathbf{\Pi} = \mathbf{B}\mathbf{A}^T$, provided the cointegrating rank of the system is R ($0 \leq R < M$),

$\mathbf{\Gamma}_s$ - short-run parameter matrices,

$\mathbf{C}$ - deterministic coefficients matrix,

$\boldsymbol{\xi}_t = [\xi_{1t} \ldots \xi_{Mt}]$ - vector of white noise disturbances.

Results of the Monte Carlo experiments indicate that the cointegration test works better after marginalisation has been determined; but it is asymptotically irrelevant, whether the overidentifying restrictions are tested before or after the model has been marginalized and its dynamic structure set (see Greenslade et al. (2000)). Therefore, in the first step the cointegrating rank was found using a standard Johansen procedure. Following that, weak-exogeneity hypotheses were tested. Because the exogeneity tests are sensitive to the cointegrating rank, the procedure was iteratively repeated. In the third step all economic restrictions were imposed to define the long-run structural model and to allow parameter estimation. In the last step, the short-run structure was found.

The starting point of the empirical analysis was a VAR model with three lags and a deterministic component. The misspecification tests were used to verify, whether the system provides a framework that is general enough to allow the extraction of the short-run effects from the data in the course of reduced rank regression; it seems that three lags are a reasonable compromise regarding the short time series anyway.

<Table 2. around here>

Results presented in Table 2 prove that residuals from all equations are not autocorrelated and support the choice of three lags. The non-normality of residuals from some equations is perceived as resulting from a huge number of parameters of unrestricted VAR and it does not cause problems as the main reason for it is excess kurtosis, for which reduced rank regression is robust (see Gonzalo (1994)). Furthermore, no ARCH effect was detected.

The cointegration rank was determined using two standard tests: the maximum eigenvalue and the trace, whose statistics are denoted λ_{max} and λ_{trace} respectively. The latter was also corrected for a small sample bias using the correction derived by Ahn and Reinsel (1990) and Reimers (1992) (λ_{trace}^{ARC}). Both asymptotic tests gave comparable results (see Table 3). However, results provided by inference based on the trace test were treated as more plausible, because Lütkepohl (2001) argues that in some cases the trace test in cointegrated systems with a number of cointegrating relations and limited samples is superior to the maximum eigenvalue test in power terms. Inference on the cointegration rank alternated with inference on the weak-exogeneity of variables (see Tables 3 and 4, respectively). Pesaran-Shin (2000) asymptotic critical values (for 5% size) were used, which are conclusive when conditional inference is being made.

<Tables 3. and 4. around here>

In the first step λ_{trace} and λ_{trace}^{ARC} suggested that at least five common trends exist, while λ_{max} implied as many as seven. The *LR* test for weak-exogeneity indicated in turn that the price index of imported materials and intermediate products, productivity in industry, value added tax, the price index of imported consumer goods, and the rate of unemployment should be treated as weakly exogenous, which necessitated another inference on the cointegration rank in a system conditional on these variables. The result of the trace test in the second iteration suggested that at least two more common trends exist and the *LR* test for weak-exogeneity indicated that wage taxes and the share of services in total consumption should be considered weakly exogenous. In the last iteration, the cointegration rank was tested for a system conditional on the seven aforementioned variables and no more common stochastic trends were found, even though the trace test statistic corrected for a short sample indicates that one more common stochastic trend may exist. However, the expected value of the trace test statistic in a short sample for a conditional system is still unknown, which questions the correction usefulness in such circumstances.

It should be particularly stressed that all variables considered weakly exogenous proved to be such. Probably the case of import prices and thus the exchange rate needs some additional comments.

The old political system collapsed in Poland in 1989, which enabled to initiate the transformation from centrally planned towards a market economy. The so-called “stabilization program” introduced several anchors. The second most important anchor, after the draconian devaluation of the Polish zloty, was the lock-up of the exchange rate against the US dollar. From January 1990, the domestic currency started to be convertible for current account transactions, which forced Polish enterprises to compete with foreign producers and to adjust to the world market conditions. In October 1991, the nominal anchor was abandoned and a ‘crawling peg’ introduced with a monthly 1.8% devaluation against the currency basket (the US dollar 45%, German mark 35%, pound sterling 10%, French and Swiss frank 5% each). The percentage monthly devaluation was reduced in mid 1993. From mid 1995 the exchange rate policy started to be more flexible, which resulted in the introduction of the crawling band. Consequently, the exchange rate was allowed to fluctuate within ±7% band around the reference rate fixed by the National Bank of Poland. Then the monthly devaluation was gradually diminished to 0.35% in 1999 and the crawling band broadened to ±15% in the same year, also the currency basket was adjusted: the euro represented 55% and the US dollar 45%. The flexible exchange rate was introduced as late as April 2000. All this explains why in the sample period the exchange rate and consequently import prices barely depended on economic developments in Poland and thus were detected as weakly exogenous in the analysed system of macro variables.

The long-run structure was identified by subjecting parameters of matrix $\mathbf{B}$ to exclusion and homogeneity restrictions. Fifty five restrictions were imposed altogether as can be deduced from (9a) – (9e) and they were not rejected by the *LR* test for over-identifying restrictions due to statistic value $\chi 2(30)=42.11$. At this stage of the analysis, matrix $\mathbf{A}$ parameters were left unrestricted. The constant terms were decomposed into those restricted to the cointegration space and unrestricted, according to the identity exploiting an orthogonal compliment to matrix $\mathbf{A}$ (see Johansen (1996)).

Tests of the marginalized system's residuals presented in Table 5 confirm that the model is acceptable. Especially strong evidence in support of this opinion is results of the multivariate versions of the *LM* test of residual autocorrelation and the Doornik-Hansen test of normality. The univariate test's outcomes are also satisfactory, however, kurtosis shows weak signs of shocks and interventions affecting prices of consumer goods and services. Worth mentioning are high R^2 values across all the equations, but particularly in those explaining wages, prices of services and consumer prices.

<Table 5. around here>

The normality of the residuals from the consecutive equation, the lack of ARCH effect and serial correlation assure that the inference made so far is valid. Besides, it enables to seek a more parsimonious structure of parameters than the existing one.

5. The structural price-wage model. Empirical results

The maximum likelihood estimates of the long-run relationships are (absolute values of *t*-statistics are bracketed under the parameters):

$$pr = 0.313 + \underset{(3.79)}{0.258}\, pm^{I} + \underset{(10.91)}{0.742}(wc - z) \tag{9a}$$

$$pc - tv = -0.071 + \underset{(2.02)}{0.137}\, pm^{F} + \underset{(12.69)}{0.863}\, pr \tag{9b}$$

$$ps = 1.095 + pc + \underset{(12.06)}{2.183}\, h \tag{9c}$$

$$p = 0.005 + \underset{(180.06)}{0.720}\, pc + \underset{(69.94)}{0.280}\, ps \tag{9d}$$

$$wc - tx - p - z = 0.387 - \underset{(15.04)}{0.045}\, U \tag{9e}$$

The results are fully interpretable in economic terms and the model allows to identify how wages induce inflation in the long-run and, conversely, how the costs of living influence wages in the presence of exogenous taxes.

The elasticity of producer prices with respect to unit wage costs (equation (9a)) and to prices of imported materials and intermediate products is 0.742 and 0.258, respectively. This is very close to the shares of wage and import costs in total costs minus value of domestic material inputs. Also elasticities of prices of consumer goods with respect to producer prices (0.863) and imported goods (0.137) are approximately the same as mean shares of, respectively, domestically produced and imported commodities in total households consumption in the sample (equation (9b)). An analogous conclusion can be drawn in relation to the equation transforming prices of consumer goods and services into the cost of living index (equation (9d)): the elasticity with respect to the services prices equals 0.280 and it is close to the share of expenditures on services in the total expenditures of households.

The estimate of real net wages sensitivity (equation (9e)) to changes in labor market being measured by a parameter associated with the unemployment rate (-0.045) does not differ from results obtained for other developed economies in Europe. The unit elasticity of real net wages with regard to the labor productivity assumption guarantees that growing labor productivity – ceteris paribus – will not generate unlimited increases in the profits of enterprises in the long run. On the other hand, the unit price elasticity of wages ensures that inflation – ceteris paribus –will not make real wages fall, which in the long run would result in an uncontrolled decline in households' real incomes.

From the construction of bridge regression between the prices of consumer goods and services it follows that a non-increasing share of services consumption in total consumption eliminates pressures on increases in services' prices (equation (9c)). Rising indirect taxes (VAT) and overheads imposed on wages accelerate inflation, since it is not possible to cushion their impacts in the long run (the relevant elasticities have unit value, equations (9b) i (9e)).

Since (8) is entirely based on stationary variables, the standard *t*-ratios as well as other classical tests retain their properties and can be used to find loading coefficients and short-run parameters that significantly differ from zero. To find as parsimonious dynamic structure of the model as possible, the iterative procedure was applied. At first, the parameters of loading matrix and the dynamics of endogenous variables were left unrestricted, while regressors with the smallest absolute values of *t*-ratios were iteratively removed from the rest of the short-run structure. Next, all statistically insignificant variables and unimportant cointegrating vectors were omitted. This led to the following (t-statistics are parenthesized):

$$\Delta pr_t = \underset{(5.18)}{0.057}\,\mathbf{ec}_{2,t-1} + \underset{(2.98)}{0.025}\,\mathbf{ec}_{3,t-1} +$$

$$+ \underset{(1.98)}{0.131}\,\Delta pr_{t-1} +$$

$$+\underset{(6.74)}{0.015}\, m93.7_t + \underset{(6.54)}{0.014}\, m95.1_t + \underset{(2.37)}{0.002}\, m8_t + \underset{(2.96)}{0.001} + u_{1,t}$$

$$\begin{aligned}\Delta pc_t = & -\underset{(3.22)}{0.034}\,\mathbf{ec}_{1,t-1} + \underset{(11.09)}{0.119}\,\mathbf{ec}_{2,t-1} + \underset{(6.02)}{1.689}\,\mathbf{ec}_{4,t-1} - \underset{(3.71)}{0.011}\,\mathbf{ec}_{5,t-1} + \\ & + \underset{(5.00)}{0.126}\,\Delta tv_t + \underset{(2.68)}{0.107}\,\Delta pc_{t-1} + + \underset{(2.36)}{0.021}\,\Delta pc_{t-2} - \underset{(3.91)}{0.037}\,\Delta pm^F_{t-2} + \\ & - \underset{(6.32)}{0.006}\, m93.7_t - \underset{(5.68)}{0.015}\, m95.7_t + u_{2,t}\end{aligned}$$

$$\begin{aligned}\Delta ps_t = & \ \underset{(3.66)}{0.006}\,\mathbf{ec}_{3,t-1} + \underset{(4.70)}{0.635}\,\mathbf{ec}_{4,t-1} - \underset{(2.59)}{0.007}\,\mathbf{ec}_{5,t-1} + \\ & + \underset{(3.31)}{0.090}\,\Delta tv_t + \underset{(5.75)}{0.265}\,\Delta pc_{t-1} + \underset{(6.40)}{0.211}\,\Delta ps_{t-1} - \underset{(2.52)}{0.012}\,\Delta tv_{t-1} + \underset{(3.29)}{0.001}\,\Delta U_{t-1} - \underset{(2.75)}{0.001}\,\Delta U_{t-2} + \\ & + \underset{(6.71)}{0.015}\, m93.7_t + \underset{(5.08)}{0.014}\, m95.1_t + \underset{(14.39)}{0.022}\, m9698.1_t + \underset{(2.21)}{0.0003}\, m4_t - \underset{(2.30)}{0.0003}\, m11_t + u_{3,t}\end{aligned}$$

$$\begin{aligned}\Delta p_t = & -\underset{(3.20)}{0.024}\,\mathbf{ec}_{1,t-1} + \underset{(11.19)}{0.085}\,\mathbf{ec}_{2,t-1} + \underset{(6.86)}{1.414}\,\mathbf{ec}_{4,t-1} - \underset{(4.42)}{0.011}\,\mathbf{ec}_{5,t-1} + \\ & + \underset{(5.67)}{0.119}\,\Delta tv_t + \underset{(4.14)}{0.0003}\,\Delta U_t + \underset{(5.91)}{0.212}\,\Delta p_{t-1} + \underset{(2.54)}{0.019}\,\Delta p_{t-2} - \underset{(3.90)}{0.027}\,\Delta pm^F_{t-2} + \\ & + \underset{(4.48)}{0.004}\, m95.1_t - \underset{(5.85)}{0.011}\, m95.7_t + \underset{(12.85)}{0.006}\, m9698.1_t + u_{4,t}\end{aligned}$$

$$\begin{aligned}\Delta wc_t = & -\underset{(10.32)}{0.505}\,\mathbf{ec}_{1,t-1} + \underset{(8.74)}{0.477}\,\mathbf{ec}_{2,t-1} + \underset{(3.42)}{6.497}\,\mathbf{ec}_{4,t-1}\ \underset{(3.18)}{0.052}\,\mathbf{ec}_{5,t-1} + \\ & + \underset{(10.00)}{0.537}\,\Delta z_t + \underset{(5.36)}{0.046}\,\Delta U_t - \underset{(4.90)}{0.065}\,\Delta U_{t-1} - \underset{(3.25)}{0.146}\,\Delta wc_{t-2} + \underset{(4.07)}{0.036}\,\Delta U_{t-2} + \\ & + \underset{(2.97)}{0.028}\,\Delta m9495.3_t - \underset{(4.66)}{0.035}\,\Delta m9698.1_t - \underset{(6.09)}{0.042}\,\Delta m0003.1_t - \underset{(3.37)}{0.015}\, m8_t - \underset{(6.55)}{0.027}\, m9_t + \\ & - \underset{(10.14)}{0.041}\, m10_t + \underset{(4.20)}{0.022} + u_{5,t}\end{aligned}$$

where $\mathbf{ec}_1$, $\mathbf{ec}_2$, $\mathbf{ec}_3$, $\mathbf{ec}_4$ and $\mathbf{ec}_5$ are cointegrating vectors as specified by (9a), (9b), (9c), (9d) and (9e), respectively. The $m4$, $m8$, $m9$, $m10$ and $m11$ are seasonal centered dummies for the appropriate months. Other variables that start with "m" are also dummies, for example $m9495.3$ takes ones in March in years 1994 and 1995.

In the first row of each equation there are cointegrating vectors (multiplied by the loading coefficients), in the second row short run dynamics (first changes of the weakly exogenous variables, lagged first and second differences of all variables), and the last row contains deterministic terms (dummies, centered seasonal dummies and constants outside cointegrating space) plus error terms.

It is worth stressing that various ways of eliminating insignificant regressors led to very similar structures.

Interestingly, in the equation of the producer prices only two cointegrating vectors identified as the long-run equations for consumer goods prices and prices of services were significant, while the cointegrating vector defining the producer prices' disequilibrium was not. In fact, this allows to state that producers are more likely to adjust themselves to pressures arising from the consumer prices than to their own prices, which can serve as indirect evidence that the Polish economy is demand driven indeed. In the remaining equations, the structure of loading coefficients is rather rich and parameters located on the diagonal of matrix $\mathbf{A}$ are non-zero, so it is hard to give the economic interpretation.

Constants outside the cointegration space proved to be statistically significant only in the equation of producer prices and wages, but even then their values are very small. This means that none of the endogenous variables shows an autonomous growth.

The determination coefficients are all satisfactory, as the behavior of the five endogenous variables growth rates was explained from 85% to 95% in individual cases**.**

6. Conclusions

Cointegration analysis enabled to build model of the price-wage behavior with nonstationary variables under the presence of direct and indirect taxes. The model meets all economic assumptions about the long-run behavior of the interesting variables. In particular, static homogeneity is implicit in both price and wage equations. Its innovation lies in the desegregation of prices and distinguishing between producer prices and the cost of living index, which turned to be crucial in appropriate modeling.

In the presence of relatively rich short-run dynamics, as many as five identified cointegrating vectors proved to be significant for explaining variations of the modeled variables. This shows that relationships between producer prices, consumer goods and services prices, and wages are very complex and must be studied within multivariate systems.

As any other model, also this one simplifies economic reality. It is based on standard, one might say, classical assumptions. Nonetheless, this system was able to explain almost all variations of the analyzed macroeconomic processes and passed all statistical tests. It confirms that the development of economies in transition can be well explained using similar models as those applied to developed European economies.

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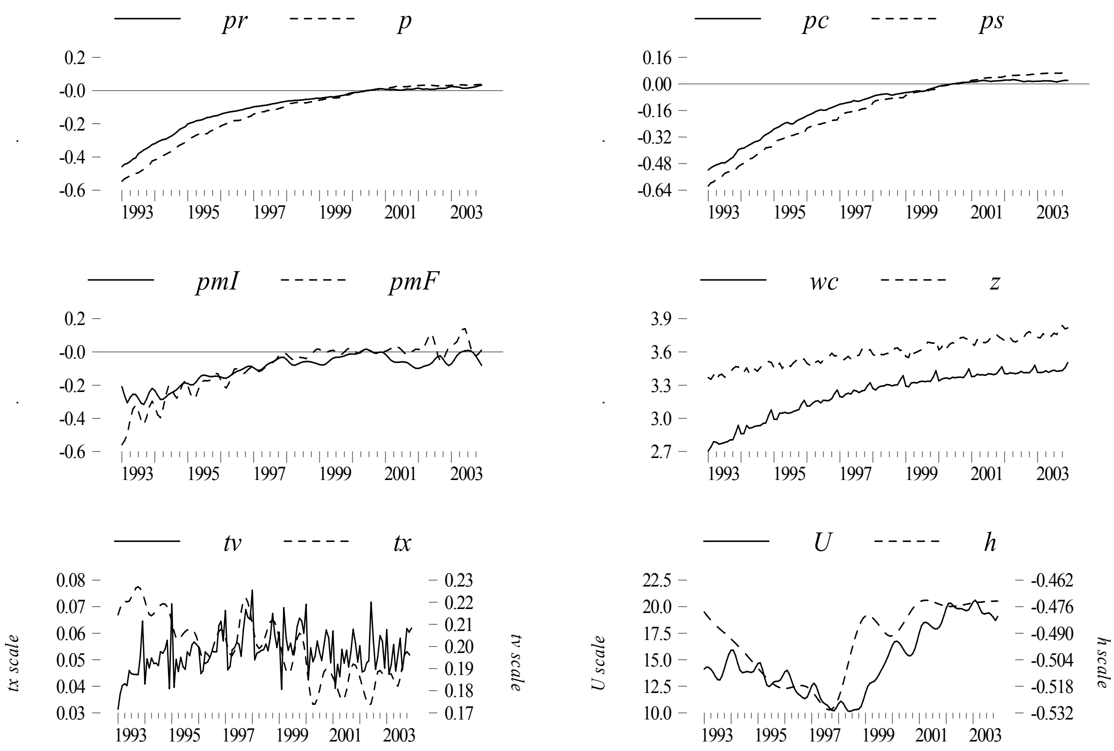

Figure 1. Seasonally unadjusted monthly data, January 1993 to December 2003

Table 1. Inference on the order of integration

variable	hypotheses	t statistic of the ADF test	LM statistic of the KPSS test	ADF test conclusion, limit distribution size - 5%	KPSS test conclusion, limit distribution size - 5%	ADF-KPSS joint test conclusion, exact distribution		K-S statistics of the cumulated periodogram test
						order of integration	probability of joint confirmation	
pr_t	I(2) vs. I(1)	-7.710	0.103	I(1)	I(1)	I(2)	0.95	0.142
pc_t		-4.789	0.146	I(1)	I(1)	I(2)	0.95	0.074
ps_t		-5.990	0.069	I(1)	I(1)	I(1)	0.90	0.091
p_t		-6.209	0.138	I(1)	I(1)	I(2)	0.95	0.076
wc_t		-7.073	0.104	I(1)	I(1)	I(2)	0.95	0.101
pm^I_t		-8.511	0.072	I(1)	I(1)	I(1)	0.90	0.167
z_t		-7.702	0.046	I(1)	I(1)	I(1)	0.99	0.124
tv_t		-7.928	0.100	I(1)	I(1)	I(2)	0.95	0.122
pm^F_t		-7.656	0.056	I(1)	I(1)	I(1)	0.95	0.064
h_t		-3.502	0.323	I(1)	I(1)	I(2)	0.95	0.134
tx_t		-6.605	0.077	I(1)	I(1)	I(1)	0.90	0.165
U_t		-4.569	0.111	I(1)	I(1)	I(2)	0.95	0.110
pr_t	I(1) vs.I(0)	-3.708	0.353	I(1)	I(1)	I(1)	0.95	0.091
pc_t		-4.171	0.372	I(0)	I(1)	I(1)	0.95	0.077
ps_t		-1.366	0.385	I(1)	I(1)	I(1)	0.95	0.105
p_t		-3.849	0.376	I(0)	I(1)	I(1)	0.95	0.089
wc_t		-2.439	0.379	I(1)	I(1)	I(1)	0.95	0.138
pm^I_t		-0.986	0.324	I(1)	I(1)	I(1)	0.95	0.073
z_t		-0.895	1.570	I(1)	I(1)	I(1)	0.95	0.106
tv_t		-4.615	0.519	I(0)	I(1)	I(1)	0.95	0.083
pm^F_t		-3.267	0.377	I(1)	I(1)	I(1)	0.95	0.059
h_t		-0.826	0.815	I(1)	I(1)	I(1)	0.95	0.075
tx_t		-1.661	1.236	I(1)	I(1)	I(1)	0.95	0.113
U_t		-0.940	0.339	I(1)	I(1)	I(1)	0.95	0.113

Table 2. Misspecification tests

Equation	K-S statistics of Durbin test	Kurtosis	Normality, $\chi2(2)$	ARCH(3), $\chi2(3)$	ARCH(12), $\chi2(12)$	R^2
pr_t	0.080	2.979	0.226	8.673	20.245	0.858
pc_t	0.125	3.516	4.243	10.345	26.400	0.891
ps_t	0.125	4.093	7.612	5.819	23.073	0.914
p_t	0.095	4.574	11.049	16.156	28.402	0.903
wc_t	0.062	3.490	3.006	10.164	21.518	0.960
$pm^I{}_t$	0.093	8.009	71.252	12.826	7.273	0.856
z_t	0.143	2.857	1.381	1.327	3.895	0.922
tv_t	0.052	7.718	33.538	0.345	4.557	0.841
$pm^F{}_t$	0.165	3.553	3.665	4.350	25.472	0.948
h_t	0.138	7.798	67.785	13.686	27.368	0.999
tx_t	0.158	3.750	4.899	12.313	20.600	0.984
U_t	0.087	6.103	26.598	9.087	22.991	0.981

Table 3. Inference on the cointegration rank

Number of common trends	λ_{max}	λ_{trace}	λ^{ARC}_{trace}
12	150.77	620.76	451.29
11	99.62	469.99	341.68
10	86.53	370.37	269.26
9	70.70	283.84	206.35
8	62.12	213.14	154.95
7	51.32	151.02	109.79
6	34.13	99.70	72.48
5	29.04	65.57	47.67
4	16.34	36.53	26.56
3	12.28	20.19	14.68
2	5.55	7.91	5.75
1	2.36	2.36	1.72
weakly exogenous variables: $pm^I{}_t$, z_t, tv_t, $pm^F{}_t$, U_t			
7	116.98	402.91	292.91
6	84.07	285.93	207.87
5	80.81	201.86	146.75
4	53.53	121.05	88.00
3	38.63	67.52	49.08
2	19.25	28.89	21.00
1	9.64	9.64	7.00
weakly exogenous variables: $pm^I{}_t$, z_t, tv_t, $pm^F{}_t$, U_t, tx_t, h_t			
5	81.91	271.49	197.37
4	70.01	189.58	137.82
3	59.87	119.57	86.93
2	31.85	59.70	43.40
1	27.85	27.85	20.25

Table 4. Inference on weak-exogeneity

variable	χ2(7)
h_t	79.59
p_t	64.14
pc_t	59.92
ps_t	45.90
tx_t	43.71
pr_t	34.52
wc_t	29.28
pm^I_t	16.66
tv_t	15.26
U_t	13.55
z_t	8.37
pm^F_t	7.76
weakly exogenous variables: pm^I_t, z_t, tv_t, pm^F_t, U_t	
variable	χ2(5)
p_t	47.54
pc_t	47.38
ps_t	43.41
pr_t	29.24
wc_t	28.20
tx_t	15.57
h_t	12.88

Table 5. Residual analysis

UNIVARIATE TESTS						
Equation	K-S statistics of Durbin test	Kurtosis	Normality, χ2(2)	ARCH(3), χ2(3)	ARCH(12), χ2(12)	R^2
Δpr_t	0.094	3.198	1.038	10.068	21.887	0.849
Δpc_t	0.080	3.987	7.121	8.966	14.214	0.897
Δps_t	0.104	3.926	6.425	7.689	21.429	0.912
Δp_t	0.077	3.956	6.566	7.785	21.593	0.917
Δwc_t	0.104	3.156	0.770	2.421	25.106	0.962
MULTIVARIATE TESTS						
residual autocorrelation LM1	χ2(25) = 31.233	ρ-value = 0.18				
residual autocorrelation LM4	χ2(25) = 18.976	ρ-value = 0.80				
normality: LM	χ2(10) = 8.243	ρ-value = 0.61				