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A risk-driven approach to exchange-rate modelling

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A Risk-Driven Approach to Exchange-Rate Modelling*

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Abstract
The paper presents a new approach to exchange rate modelling that augments the CHEER model with a sovereign credit default risk as perceived by financial investors making their decisions. In the cointegrated VAR system with nine variables comprised of the short- and long-term interest rates in Poland and the euro area, inflation rates, CDS indices and the zloty/euro exchange rate, four long-run relationships were found. Two of them link term spreads with inflation rates, the third one describes the exchange rate and the fourth one explains the inflation rate in Poland. Transmission of shocks was analysed by common stochastic trends. The estimation results were used to calculate the zloty/euro equilibrium exchange rate.

JEL classification numbers: C32, E31, E43

Keywords: exchange rate modelling, sovereign credit default risk, CDS spread, international parities, equilibrium exchange rate

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I. Introduction

One of the consequences of the 2007 financial crisis has been uncertainty and risk growing in the financial markets. Risk has certainly become one of the more significant factors in the determination of exchange rates. In the short and medium-run, it is the risk level subjectively assigned to a country and the interest rate levels set by the country's monetary authorities that determine the inflow of foreign capital and thereby the exchange rate.

This empirical study focuses on the Polish economy. Being a medium-sized country and an EU member since 2004, Poland still remains outside the ERM2 mechanism which implies the pegged float exchange rate regime with a margin of 15%, however promised to join Monetary Union. The floating of the country's currency (the zloty) was officially introduced on 12 April 2000, but the fixed exchange rate was repealed a year before, on 7 June 1999. The last adjustments under the crawling-band mechanism were made in 1998. This means that the floating exchange rate began in 1999.

In the case of new EU members the exchange-rate modelling approaches that are typically applied to large and developed economies and long time spans present limited usefulness. Therefore this study starts with the capital enhanced equilibrium exchange rate (CHEER) hypothesis which is aimed at medium-run and rather small open economies. The CHEER approach highlights the capital account in the balance of payments condition. Therefore it assumes that the real exchange rate follows movements of the interest rates and prices differentials, whereas the determinants of the current account are ignored. The CHEER model was augmented with sovereign credit default swaps in order to measure risk perceived by financial investors. This approach proved to be extremely important for explaining the exchange rate's behaviour before and during the financial crisis.

The paper is organized as follows. Section two provides the specification of the exchange rate model. The next section presents the cointegrated VAR model and the empirical results it generated. The impacts of the stochastic shocks and their transmission channels are analysed in section four. The equilibrium exchange rate is calculated in section five. The paper ends with conclusions.

II. The behaviour of the exchange rate and the credit default risk

The CHEER approach to exchange rate modelling follows from the combination of the purchasing power parity (PPP) and the uncovered interest rate parity (UIP) hypotheses (Johansen and Juselius, 1992; Juselius, 1995; MacDonald, 2000). Adding both short- and long-term interest rates and inflation rates offers a much superior explanation of the behaviour of the exchange rate, as well as enabling the verification of a number of other theories, including the Fisher decomposition, the term structure (TS), and the real interest rate parity (RIP) (Juselius and MacDonald, 2003, 2004; Kębłowski and Welfe, 2010):

$$\left(i_t^l - ie_t^l\right) + \omega_1\left(i_t^s - ie_t^s\right) + \omega_2\left(\Delta p_t - \Delta pe_t\right) + \omega_3 q_t = \varepsilon_t , \qquad (1)$$

where i_t^m and ie_t^m denote interest rate yields inside and outside the country, respectively (m=l for the long run and m=s for the short run); p_t and pe_t are the logs of consumer price indexes inside and outside the country; q_t is the real exchange rate; $q_t = p_t - pe_t - s_t$, s_t is the log of the spot exchange rate; ω_1, ω_2 and ω_3 are weights and ε_t is a (weakly) stationary error term.

The interest rate yield can be decomposed into a real interest rate and an expected inflation rate (the Fisher decomposition):

$$\begin{aligned} i_t^m &= r_t^m + \Delta_m p_{t+m}^e / m , \\ ie_t^m &= re_t^m + \Delta_m pe_{t+m}^e / m , \end{aligned} \qquad (2)$$

where r_t^m and re_t^m stand for real interest rates inside and outside the country, respectively. If the short-term inflationary expectations are naïve rather than rational (Łyziak, 2003) and the expectations of long-term inflation are equal to the long-term inflation target, the above equations can be rewritten as follows:

$$\begin{aligned} i_t^s &= r_t^s + \Delta p_t , \\ i_t^l &= r_t^l + c, \end{aligned} \tag{3}$$

where c denotes a constant (a long-term inflation target). This implies that if $\omega_1 < 0$, then $\omega_2 \geq 0$.

The above combined with the TS hypothesis gives:

$$\begin{aligned} i_t^s - i_t^l &= \Delta p_t + \left(r_t^s - r_t^l \right) - c, \\ ie_t^s - ie_t^l &= \Delta pe_t + \left(re_t^s - re_t^l \right) - ce. \end{aligned} \tag{4}$$

which means that if the differentials between the short- and long-term real interest rates are stationary, then the term spreads depend on the rates of inflation and (probably negative) constants.

The RIP hypothesis states that the real interest rate differential should be stationary. The RIP combined with the Fisher decomposition produces:

$$\begin{aligned} r_t^s - re_t^s &= \left(i_t^s - ie_t^s \right) - \left(\Delta p_t - \Delta pe_t \right), \\ r_t^l - re_t^l &= \left(i_t^l - ie_t^l \right) - \left(c - ce \right). \end{aligned} \tag{5}$$

Interestingly, when an assumption about long-term inflationary expectations being constant is made, then the RIP hypothesis implies stationarity of the differential of the nominal long-term interest rates up to some constant.

The turmoil in the global financial market in late 2008 revealed that the earlier models ommited some important factor. Rapidly changing risk aversion and capital outflows caused that exchange rates depreciated at the same pace in many emerging markets. This may suggest that financial investors perceive risk premium as an important force driving exchange rates.

There are different measures of risk, such as a real exchange rate denominated in US dollars for the assumed safety of this currency, an equity market variable (for similar reasons), liquidity indices, the term spread, the term spread denominated in foreign currency, collateralized debt obligations and other over-the-counter derivatives. However, most financial market players use the index of credit default swaps (CDS) for governmental bonds, despite some concerns about the concentration of the over-the-counter derivative markets (ECB report, 2009). The quotation of the sovereign CDS index can pass for a risk premium, if no counterparty risk is involved in the party selling the protection (or if the risks are mutually independent) and assuming that the market for this derivative is broad and liquid enough.

The effect of the sovereign credit default on the behaviour of the exchange rate is accounted for with the following equation:

$$\left(i_t^l - ie_t^l\right) + \omega_1\left(i_t^s - ie_t^s\right) + \omega_2\left(\Delta p_t - \Delta pe_t\right) + \omega_3\left(cds_t - cdse_t\right) + \omega_4 q_t = \varepsilon_t , \tag{6}$$

where cds_t and $cdse_t$ denote the logs of the CDS indices within and outside the country, respectively; $\omega_1, \ldots ,\omega_4$ are the weights. The normalization of the above equation with respect to q_t produces an exchange rate model which will be subjected to empirical verification.

The market for the credit default swaps on government bonds is growing. For the first time Polish bonds were quoted in November 2000. The CDS index for German bonds (representing the foreign counterpart of the Polish bonds) has been quoted since March 2003. Because both series are significantly correlated with the equity market variable, they were backdated to January 1999 to make sure that the sample was sufficiently long for analysis. The estimates of these auxiliary reduced rank regressions are $cdse_t = \underset{(8.2)}{13.65} - \underset{(2.7)}{1.443}\, dax_t + \underset{(3.8)}{0.003}\, t$ and $cds_t = \underset{(6.4)}{10.69} - \underset{(3.1)}{1.014}\, wig20_t - \underset{(2.9)}{0.021}\, t$, where dax_t and $wig20_t$ denote the logs of the stock market indices. The series are illustrated in Figure 1. The first half of the sample shows that in particular years before 2005 Polish bonds were usually perceived as less risky compared with their foreign counterpart and the opinion was probably connected with the country's plans to join the European Union. The most noticeable, though, are the huge increments in the quotations of both countries' CDS at the end of 2008, when financial markets plunged into turmoil. As a matter of fact, that risk was increasing was signalled for first time in early 2008 and since that year onwards Polish bonds were treated as riskier than before (see the lower graph in Figure 1). Interestingly, the last graph clearly shows that the shock Poland and the euro area received at the end of 2008 was actually the same.

<insert Figure 1 here>

III. The long-run relationships

Considering that the system's variables are integrated of order one (see Figure A1 and Table 1), the transmission mechanisms can be inferred from the VEC model

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta x_{t-j} + \delta D_t + \varepsilon_t \,, \tag{7}$$

if the reduced rank restriction holds. Here, α and β are full-rank matrices, the first being the loadings matrix and the second containing the coefficients of the cointegrating vectors; D_t is a vector of deterministic variables and $x_t = \begin{bmatrix} q_t & \Delta p_t & \Delta pe_t & i_t^l & ie_t^l & i_t^s & ie_t^s & cds_t & cdse_t \end{bmatrix}'$. The variables are defined as follows: q_t is the real exchange rate; Δp_t and Δpe_t denote the CPI-based deseasonalized inflation rates in Poland and the euro area, respectively; i_t^l, ie_t^l, i_t^s, ie_t^s are monthly average yields on five-year floating-rate bonds and three-month interbank offered rates (monthly yield) in Poland and the euro area; cds_t and $cdse_t$ are the logs of the sovereign credit default swap indices for Poland and Germany, Germany being used as a euro area substitute. Because of the floating exchange rate regime, the monthly series start in January 1999. The last observation in the sample comes from April 2010. The data were derived from the official sources of the Polish Statistical Office (GUS) and EUROSTAT.

<insert Table 1 here>

In line with the multivariate LM test for consecutive residual autocorrelations and information criteria, the lag length was set to $k = 2$, so that the probability values for the first and second order residual autocorrelation equalled 0.10 and 0.14, respectively. The Doornik-Hansen (2008) test for multivariate normality rejected the null hypothesis, mainly due to excess kurtosis. The Monte Carlo experiments showed, however, that a cointegration analysis based on small samples is robust to excess kurtosis (Gonzalo, 1994) and to some non-normal distributions of innovations (Kębłowski, 2005). Besides, the recursive estimation suggested that in the period in question structural breaks did not affect the data. Hence, the only deterministic variable is the constant restricted to the cointegration space.

The estimated eigenvalues and the LR test for cointegration rank of the VEC model are reported in Table 2. Four eigenvalues seem at a glance to be clearly and significantly different from zero. Both LR tests compared with the asymptotic distributions confirm this conjecture. By contrast, the trace test with a Bartlett correction suggests that only three non-zero eigenvalues are present, but the cointegration rank test applied to small samples and high-dimensional systems is usually less powerful. Moreover, the Bartlett correction frequently tends to slightly overestimate the test size (Johansen, 2002). Further, the LR test of weak exogeneity clearly indicates that only the long-term interest rates are weakly exogenous in the euro area. Therefore, because ie_t^l is weakly exogenous, the inference based on the conditional VEC model leads to more efficient estimates:

$$\Delta y_t = (\alpha_1 - \Upsilon\alpha_2)\beta' x_{t-1} + \sum_{j=1}^{k-1}(\Gamma_{j1} - \Upsilon\Gamma_{j2})\Delta x_{t-j} + \Upsilon z_t + (\delta_1 - \Upsilon\delta_2)D_t + \tilde{\varepsilon}_{1t}, \quad (8)$$

where $x_t = [q_t \quad \Delta p_t \quad \Delta pe_t \quad i_t^l \quad i_t^s \quad ie_t^s \quad cds_t \quad cdse_t]'$, $\Upsilon = \Omega_{12}\Omega_{22}^{-1}$, $\tilde{\varepsilon}_{1t} = \varepsilon_{1t} - \Upsilon\varepsilon_{2t}$, while the marginal model is:

$$\Delta z_t = \alpha_2\beta' x_{t-1} + \sum_{j=1}^{k-1}\Gamma_{j2}\Delta x_{t-j} + \delta_2 D_t + \varepsilon_{2t}, \quad (9)$$

where $z_t = ie_t^l$ and $\alpha_2 = 0$. The LR cointegration rank test for model (8) points to four cointegrating vectors being present. The corresponding probability value of the size-corrected test is essentially smaller, although still above the usually accepted levels. The confidence bands for the recursively estimated eigenvalues additionally suggest that the system has four cointegration vectors. This allows concluding that $r = 4$.

<insert Table 2 here>

In order to identify the long-run structure, the $\dim(\mathrm{sp}(B) \cap \mathrm{sp}(H)) \geq r_1$ hypotheses where matrix H defines the independent restrictions were tested for $r_1 = 1$, which means that some restrictions were imposed on the single cointegrating vector, while other vectors remained unrestricted – $B = (H\varphi \mid \psi)$ (Johansen and Juselius, 1992). The results shown in Table 3 that were obtained for the hypotheses about long-run relationships postulated by the theory can be summarized as follows.

<insert Table 3 here>

Firstly, $\mathcal{H}_1$ to $\mathcal{H}_4$ assume that the real long- and short-term interest rates ($\mathcal{H}_1$ to $\mathcal{H}_4$) are stationary. This assumption was clearly rejected.

Secondly, the LR test supports the real long-term interest rate parity ($\mathcal{H}_5$), but the real short-term interest rate parity ($\mathcal{H}_6$) is rejected. It is noteworthy, though, that the

$\mathcal{H}_5$ hypothesis becomes unacceptable, if the sample is extended to the years prior to 1999 (see the real interest rate graphs in Figure A2). This result is a recommendation for undertaking further investigations, which make use of a longer time span.

Thirdly, both term spreads as well as their parity are rejected ($\mathcal{H}_7$ to $\mathcal{H}_9$).

Fourthly, the LR test accepts the unrestricted relationship between the term spread and the inflation rate in Poland ($\mathcal{H}_{10}$) and the euro area ($\mathcal{H}_{12}$), as well as the relationships with imposed homogeneity restriction. Therefore, the relations presented in (4) are supported, if the differentials of the short- and long-term real interest rates are stationary.

Fifthly, contrary to the results in Kębłowski and Welfe (2010), the incorrect coefficients prevent us from accepting the relationship between the real exchange rate and the parity of the real interest rates ($\mathcal{H}_{14}$ and $\mathcal{H}_{15}$). The most probable reason for which the coefficients are incorrect is the lack of significant exchange rate determinants as suggested by (6). With the inclusion of the differential of the sovereign credit default swap indices ($\mathcal{H}_{16}$, $\mathcal{H}_{17}$ and $\mathcal{H}_{19}$) the relationship between the real exchange rate and the parity of the nominal interest rates ($\mathcal{H}_{19}$) can be restored.

Finally, the test strongly supports the existence of a homogenous relationship between the Polish and the euro area inflation rates and the long-term interest rate in Poland ($\mathcal{H}_{20}$), which follows the findings presented by Juselius and MacDonald (2003, 2004), as well as Kębłowski and Welfe (2010).

We tested next whether the joint hypothesis $\mathcal{H}_{21} : B = \left(H_1\varphi_1 \mid H_2\varphi_2 \mid H_3\varphi_3 \mid H_4\varphi_4\right)$ was supported. In the hypothesis, matrices H_1 and H_2 correspond to the homogeneous relationship between the term spread and the inflation rate in Poland ($\mathcal{H}_{11}$) and the euro area ($\mathcal{H}_{13}$), respectively; H_3 stands for the relationship between the real exchange rate and the differentials of the nominal interest rates; the sovereign credit default swaps indices ($\mathcal{H}_{19}$) and H_4 describe the relationship between the Polish and euro area inflation rates and the long-term interest rate in Poland ($\mathcal{H}_{20}$). The LR test of 17 overidentifying restrictions accepted the joint hypothesis $\mathcal{H}_{21}$ with probability value of 0.29.

As the next step, the parameters of the conditional VEC model were estimated for $\mathcal{H}_{21}$, which produced the following results (the absolute values of the standard deviations of the parameter estimates are given in the brackets, $ec_{\bullet t}$ denotes weakly exogenous error correction terms):

$$\left(i_t^s - i_t^l\right) = \Delta p_t - \underset{(8.54)}{0.002} + ec_{1t}, \tag{10a}$$

$$\left(ie_t^s - ie_t^l\right) = \Delta pe_t - \underset{(16.49)}{0.002} + ec_{2t}, \tag{10b}$$

$$q_t = \underset{(2.50)}{25.540}\left(i_t^l - ie_t^l\right) - \underset{(3.76)}{0.129}\left(cds_t - cdse_t\right) - \underset{(26.02)}{1.206} + ec_{3t}, \tag{10c}$$

$$\Delta p_t = \underset{(6.90)}{0.323}\Delta pe_t + 0.677 i_t^l - \underset{(5.67)}{0.002} + ec_{4t}. \tag{10d}$$

The first two equations connect the yield gaps with the inflation rates in Poland and the euro zone, respectively. As already shown, contrary to the term structure hypothesis, the term spreads are nonstationary and apparently cointegrate with the inflation rates (see Figure A2). The interpretation of these relationships is the following. The long-term inflationary expectations driving the long-term interest rates are comparable with the constant long-run term inflation target. The short-run inflationary expectations are mostly guided by the current price changes. Therefore, inflation rates can well explain the variation in the term spreads with respect to the long-term inflation target. Moreover, the constants in these equations can be easily decomposed into a fixed long-term inflation target and an average value of the differential of the long- and short term real interest rates, see equation (4).

The third equation describes the relationship between the real exchange rate, the long-term interest rate spread and the sovereign CDS spread. Thus, the nominal exchange rate depends on price spreads, long-term interest rates and risk premiums assigned to individual economies. Unlike the first two spreads that are commonly used to explain the spot exchange rates (for example Inci and Lu, 2004; Chen and Chou, 2010) the CDS index spread has been used as a measure of risk for the first time, although this indicator is a popular decision-making tool among financial investors. The equation shows that when the risk level assigned to the Polish economy increases, the exchange rate depreciates, as apparently happened at the turn of 2008. The results indicate that, *ceteris paribus,* the doubling of the risk (a 100% increase) would lead to almost 13% depreciation of the zloty in the long-run.

The last equation presents the homogenous relationship between the inflation rate in Poland, the inflation rate in the euro zone and the long-term interest rates in Poland. The interpretation is straightforward: the Polish inflation rate follows the inflation rate "imported" from the euro zone (because the EMU countries are the major players in the Polish foreign trade) and the long-term interest rate approximating the costs of capital.

All four identified cointegrating vectors that represent the system's long-run relationships have full economic interpretation. They prove that the exchange rate, inflation and interest rates are highly interdependent and that the risk as measured by the credit default swaps significantly determines the long-run behaviour of the exchange rate.

Table 4 shows the estimated loadings matrix $\hat{\alpha}_1$. As expected, the real exchange rate is affected by the third cointegrating vector and, additionally, by the first relationship:

$$\Delta q_t = -4.71\left(i_{t-1}^s - i_{t-1}^l - \Delta p_{t-1} + 0.002\right) +$$
$$-0.09\left(q_{t-1} - 25.540\left(i_{t-1}^l - ie_{t-1}^l\right) + 0.129\left(cds_{t-1} - cdse_{t-1}\right) + 1.206\right) +$$
$$+ \ short\text{-}run \ \ terms.$$

Accordingly, any imbalances between the nonstationary behaviour of the term spread and the inflation rate in Poland, and between the real exchange rate, the long-term interest rate spread and the sovereign CDS spread pull the real exchange rate towards a new steady-state. It is worth noting that the loading value of 0.09 suggests that the differential between the exchange rate and its equilibrium value takes about seven months to decrease by half, which means that the real exchange rate will need many months to regain its steady-state after it departed from the equilibrium path.

<insert Table 4 here>

The inflation rate in Poland adjusts to the third cointegrating vector and the fourth vector describing the inflation rate determinants, with the half-life period being two months:

$$\Delta p_t^2 = -0.01\left(q_{t-1} - 25.540\left(i_{t-1}^l - ie_{t-1}^l\right) + 0.129\left(cds_{t-1} - cdse_{t-1}\right) + 1.206\right) +$$
$$-0.27\left(\Delta p_{t-1} - 0.323\Delta pe_{t-1} - 0.677 i_{t-1}^l + 0.002\right) + \ldots.$$

On the other hand, the inflation rate in the euro area follows the second vector, thus defining the relationship between the term spread and the inflation rate in the euro area:

$$\Delta pe_t^2 = +0.82\left(ie_{t-1}^s - ie_{t-1}^l - \Delta pe_{t-1} + 0.002\right) + \ldots.$$

The interest rates in Poland and the euro area are influenced by the first and the second vector, respectively:

$$\Delta i_t^l = 0.1\left(i_{t-1}^s - i_{t-1}^l - \Delta p_{t-1} + 0.002\right) +$$
$$0.11\left(\Delta p_{t-1} - 0.323\Delta pe_{t-1} - 0.677 i_{t-1}^l + 0.002\right) + \ldots,$$

$$\Delta i_t^s = -0.05\left(i_{t-1}^s - i_{t-1}^l - \Delta p_{t-1} + 0.002\right) + \ldots,$$

$$\Delta ie_t^s = -0.02\left(ie_{t-1}^s - ie_{t-1}^l - \Delta pe_{t-1} + 0.002\right) + \ldots .$$

This means that any imbalances occurring between the long- and short-term interest rates and the inflation rates affect the former, despite the loadings being small and borderline significant. The long-term interest rate in Poland follows the fourth cointegrating vector rather fast.

Finally, both sovereign credit default swap indexes are influenced by the third vector representing the relationship between the exchange rate and its determinants, see (10c).

The results prove that the dynamic behaviour of the system's variables is fairly complex. This means that economic hypotheses involving two or three economic categories bundled together within the framework of standard regression oversimplify real behaviour, fall short of explaining the observed phenomena and most probably lead to false conclusions.

IV. The underlying driving forces

Once the long-run relationships are known, the forces pulling the variables towards long-run equilibrium can be identified. An equally interesting research issue is trying to find the nonstationary forces that push the whole system. With this end in view, the VAR model was inverted into a moving average. For the I(1) variables the MA form can be rewritten as follows:

$$x_t = C\sum_{i=1}^{t}\left(\varepsilon_i + \delta D_i\right) + C_1(L)\left(\varepsilon_t + \delta D_t\right) + A, \tag{11}$$

where $C = \beta_\perp\left(\alpha_\perp' \Gamma \beta_\perp\right)^{-1}\alpha_\perp'$, $\Gamma = I - \sum_{i=1}^{k-1}\Gamma_i$, $\alpha_\perp$ and $\beta_\perp$ are matrices of full rank and dimension $p \times (p-r)$, such that $\alpha'\alpha_\perp = \beta'\beta_\perp = 0$; $C_1(L)$ is an infinite-order polynomial given by the parameters of the VAR model; A depends on the initial values and $\beta' A = 0$ (Johansen, 1996, p. 49).

The focus is on the $\alpha_\perp$ matrix, as only the $p-r$ linear combinations of the cumulative sums of innovations enter into the process generating x_t. Thus, $\alpha'_\perp \sum_{i=1}^{t} \varepsilon_i$ defines the common stochastic trends representing the forces that drive the system in the long-run, whereas the $\tilde{\beta}_\perp = \beta_\perp \left(\alpha'_\perp \Gamma \beta_\perp\right)^{-1}$ matrix describes the loadings to the common trends. The C matrix displays the long-run effect of cumulated disturbances on the variables. Further, the $C_1(L)$ matrix represents the short-run effects generated by shocks.

Table 5 shows the $\hat{\alpha}'_\perp$, normalized $\hat{\alpha}'_\perp$ and $\hat{\tilde{\beta}}'_\perp$, $\hat{C}$ matrices and the residual standard deviation for the restrictions imposed on β under the joint hypothesis $\mathcal{H}_{21}$. It is very evident that the greatest coefficients in matrix $\hat{\alpha}_\perp$ correspond to the interest rates and the inflation rate in the euro area. However, although the coefficient of $\hat{\varepsilon}_{\Delta pe}$ is 23 times greater than that of $\hat{\varepsilon}_{cds}$, its residual standard deviation is greater by more than one thousand times. Likewise, the coefficient of $\hat{\varepsilon}_{i^l}$ is about 16 times greater than the coefficient of $\hat{\varepsilon}_q$, but the residual standard deviation of $\hat{\varepsilon}_{i^l}$ is greater by as many as 43 times. As a result, the following shocks were chosen for normalization: $\hat{\varepsilon}_q$, $\hat{\varepsilon}_{ie^l}$, $\hat{\varepsilon}_{i^s}$, $\hat{\varepsilon}_{ie^s}$, $\hat{\varepsilon}_{cds}$.

<insert Table 5 here>

The cumulated shocks to inflation rates do not seem to have a significant effect on the other variables in the system, but the cumulated shocks to interest rates affect prices, as matrix $\hat{C}$ shows. In other words, in a system of variables derived from the financial markets, inflation rates follow and not push, which contrasts with the usual expectations based on the Fisher decomposition (similar results in Juselius and MacDonald, 2003, 2004). Inflation rates are influenced by the shocks to the interest rates, because of the cost of capital. In fact, the overwhelming majority of companies are incapable of carrying out their main investments without bank credit. Therefore, under the assumption about mark-up being constant in the long-run, prices should be sensitive to the shocks to the interest rates, as found.

The first common trend identified in the system is $\hat{\alpha}'_{\perp,1}\sum_{i=1}^{t}\hat{\varepsilon}_i = \sum_{i=1}^{t}\hat{\varepsilon}_{cds,i} - 0.95\sum_{i=1}^{t}\hat{\varepsilon}_{cdse,i}$. It approximates the difference between the cumulated shocks to the CDS indices for Poland and the euro area. This common trend negatively and significantly affects the real exchange rate and the long-term interest rates in the euro area, while having a positive and significant effect on both CDS indexes. The most important finding in this case is that the difference between the cumulated shocks to the CDS indexes seems to be driving the real exchange rate, which makes the spot exchange rate either depreciate or appreciate, when it is respectively positive or negative.

The next two common trends are the cumulated shocks to the short-run interest rates, which suggests that these variables are weakly exogenous and that the borderline significant loadings in the equations for the short-run interest rates are in fact insignificant. However, the *LR* test of weak exogeneity does not support these conjectures, though, most likely because of poor performance in short samples.

The cumulated shocks to the short-run interest rates positively and significantly influence the inflation rates and the short-run interest rates. Interestingly, unlike the short-run interest rate in the euro area, these common trends have a major effect on the long-run interest rate in the Polish economy. The $\hat{C}$ matrix shows that the shocks to the long-run interest rate in Poland do not significantly determine the system in the long-run, which suggests that the cumulated shocks to the short-run interest rate drive the long-run interest rate following the term structure hypothesis, even though the term spread itself is not stationary. On the other hand, the long-run interest rate in the euro area is heavily exposed to its own shocks, because the coefficients of the cumulated $\hat{\varepsilon}_{i^l}$ and $\hat{\varepsilon}_{cdse}$ in matrix $\hat{C}$ that are borderline significant seem to lose significance when restrictions are imposed on α. This is congruent with the indication of the weak exogeneity of the long-run interest rate in the euro area. According to the finding, the shocks to the long-run interest rate in the euro area represent a fourth common shock, because the coefficient of $\hat{\varepsilon}_{i^l}$ in $\hat{\alpha}'_{\perp}$ is borderline significant. This common trend has a long-run effect on the inflation rate in the euro area and on itself.

Finally, the last common trend $\hat{\alpha}'_{\perp,5}\sum_{i=1}^{t}\hat{\varepsilon}_i = \sum_{i=1}^{t}\hat{\varepsilon}_{q,i} + 0.17\sum_{i=1}^{t}\hat{\varepsilon}_{cdse,i}$ shows that a positive shock generated by the real exchange rate or the CDS index in the euro area may either bring down the inflation rate and the interest rates in Poland or increase the CDS index in the euro area.

Table 6 shows the results obtained for restrictions imposed on β according to the joint hypothesis $\mathcal{H}_{21}$ and α, i.e. assuming weak exogeneity of the long-run interest rate in the euro area. These and the previous findings are basically the same, the only exception being the loadings to the first common trend that are borderline significant for the long-run interest rate in the euro area. Therefore, when the insignificant coefficients and stationary components are left out, the MA form becomes:

$$\begin{bmatrix} q_t \\ \Delta p_t \\ \Delta pe_t \\ i_t^l \\ ie_t^l \\ i_t^s \\ ie_t^s \\ cds_t \\ cdse_t \end{bmatrix} = \begin{bmatrix} -0.121 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.699 & 0 & -0.011 \\ 0 & 2.535 & 0 & -1.475 & 0 \\ 0 & 0 & 1.074 & 0 & -0.013 \\ 0 & 0 & 0 & 1.431 & 0 \\ 0 & 0 & 1.773 & 0 & -0.023 \\ -0.001 & 2.246 & 0 & 0 & 0 \\ 2.147 & 0 & 0 & 0 & 0 \\ 1.328 & 0 & 0 & 0 & 11.226 \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} \varepsilon_{cds,i} - 0.97 \sum_{i=1}^{t} \varepsilon_{cdse,i} \\ \sum_{i=1}^{t} \varepsilon_{ie_i^s} \\ \sum_{i=1}^{t} \varepsilon_{i_i^s} \\ \sum_{i=1}^{t} \varepsilon_{ie_i^l} \\ \sum_{i=1}^{t} \varepsilon_{q,i} + 0.17 \sum_{i=1}^{t} \varepsilon_{cdse,i} \end{bmatrix} + \ldots \quad (12)$$

<insert Table 6 here>

It is worth noting that the underlying structure generating the long-run behaviour of the series is relatively straightforward, because the loadings matrix of the common trends is sparse. Besides, most of the loadings are related to the first and the fifth common trends representing the cumulated shocks to the exchange rate and the sovereign credit default swap indices.

V. The equilibrium exchange rate

The long-run structure allows estimation of the exchange rate equilibrium. To this end, the relationships (10c) will be rearranged in the following way:

$$s_t = p_t - pe_t - 25.540\left(i_t^l - ie_t^l\right) + 0.129\left(cds_t - cdse_t\right) + 1.206 - ec_{3t}, \quad (13)$$

so that the equilibrium level of the exchange rate depends on the steady-states of its determinants. One possibility is setting the determinants at average sample values and then the static equilibrium level of the spot exchange rate equal to 4.07 is obtained. This is roughly the same result as that generated in Kębłowski and Welfe (2010), however the sample did not take into account the financial crisis. This is what should be expected: financial market turmoil did not disturb the exchange rate equilibrium, which is characteristic of the long-run. This empirical outcome is certain when the processes affecting the exchange rate both prior and during the financial crisis are properly specified, i.e. when a measure of risk has been included.

The easiest way to estimate the dynamic equilibrium of the exchange rate using equation (13) is to assume that the actual prices, interest rates and CDS indexes well approximate the steady-states of the spot exchange rate and that the error term is zero. The paths of the spot exchange rate and its equilibrium levels are compared in Figure 2.

<insert Figure 2 here>

As expected, the path of the estimated equilibrium seems to be much more stable than the observed variable which shows major deviations from its level of equilibrium. In fact, considerable misalignments appear in three periods. The first significant overvaluation of the zloty started in 2001 and continued for two years. The misalignment, 11% on average, reached its peak value of 19% in the middle of the sample. In the next years, the huge depreciation of the zloty made the currency undervalued until the end of 2004. The misalignment being on average 10% reached its maximum of 20% in April 2004. Finally, the second period of zloty overvaluation that started at the end of 2007 was disrupted by the financial crisis of 2008, implying increases in both the sovereign credit default swap differential (Poland is still perceived as an emerging market) and the long-term interest rate difference. Because of the changing exchange rate determinants, the equilibrium value of the exchange rate is still relatively stable. Between September 2007 and late 2008 the misalignment was 13% on average, reaching its maximum value of 24% in August 2008.

To make the study complete, the estimated path of the equilibrium exchange rate and the time spans in which the zloty was misaligned should be compared with the results of the analyses based on other approaches. All such comparisons are made difficult, though, because the available analyses differently define the exchange rate (including the basket of currencies and price indices) and because recalculation of their results is not possible. Rubaszek (2004a) who used the FEER approach found that the zloty was overvalued between 1999 and the end of 2002, which is different from our findings. The CHEER-based analysis conducted by Kębłowski and Welfe (2010) suggests that overvaluation took place between mid-2001 and mid-2002, and then in 2006. Between the mid-2003 and the end of 2004, the zloty was undervalued. This means shorter time spans when the zloty showed misalignments.

By contrast, BEER approach applications have led to conflicting conclusions stating that the zloty was permanently overvalued between 1998 and 2003 (Rahn, 2003), that overvaluation appeared between 2000 and 2001 (Rubaszek, 2004b), or that it started in 2001 and continued for the next six quarters (Bęza-Bojanowska, 2009) and Kelm (2010)), the last conclusion being the closest to our findings. Interestingly, Bęza-Bojanowska (2009) and Kelm (2010) obtained results similar to ours also for the next two time spans of the zloty's misalignments mentioned above. However, the values of the misalignments are considerably dissimilar and so the paths of the estimated spot exchange rate equilibrium are different.

It is quite natural for all the studies to be different, because they define variables differently, use time spans of various lengths and prefer different models. Hopefully, the constantly growing number of empirical studies will facilitate the consensus over the exchange rate misalignments.

VI. Conclusions

This paper discusses the CHEER hypothesis augmented with risk as perceived by financial investors making their decisions. We have found that in addition to price and interest rate differentials the sovereign credit default risk also significantly determines the exchange rate. Four long-run relationships have been identified: two connecting the term spreads with the inflation rates, one characterizing the behaviour of the exchange rate and one describing the inflation rate in Poland.

With a system of nine variables we have shown that two of the five common trends are cumulated shocks to the exchange rate and sovereign credit default swap indices. These two common trends exert also a primary influence on the system's variables, as most loadings are related to them.

The approach we propose seems to meet well the demands of the medium-run estimations of the equilibrium exchange rate. Because of that, it may be found useful by the policy makers in the New Member States who think of bringing their countries into the EMU.

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TABLE 1

Inference on the integration order

	Δq	$\Delta^2 p$	$\Delta^2 pe$	Δi^l	Δie^l	Δi^s	Δie^s	Δcds	$\Delta cdse$
ADF	-5.25	-8.16	-8.61	-4.59	-5.10	-3.19	-3.82	-4.92	-4.85
t-test	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
KPSS	0.07	0.03	0.02	0.11	0.17	0.16	0.18	0.18	0.12
test	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
	q	Δp	Δpe	i^l	ie^l	i^s	ie^s	cds	$cdse$
ADF	-0.67	-2.09	-2.04	-1.08	-0.66	-1.25	-0.92	-0.26	0.41
t-test	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)	I(1)
KPSS	0.51	0.73	0.10	1.89	1.16	2.02	0.35	0.41	1.44
test	I(1)	I(1)	I(0)	I(1)	I(1)	I(1)	I(0)	I(0)	I(1)

Notes: Non-zero expected values of variables, without linear trend in the levels, four lags in the regression of the ADF test, four lags in the Bartlett kernel in the KPSS test.

TABLE 2

Cointegration and weak exogeneity

	cointegration rank								
$p-r$	9	8	7	6	5	4	3	2	1
λ_i	0.46	0.44	0.37	0.30	0.14	0.11	0.07	0.03	0.02
λ_{max}	178.4	145.3	95.1	67.4	33.0	20.6	12.2	4.7	2.0
λ_{trace}	323.8 (0.00)	240.4 (0.00)	162.4 (0.00)	100.4 (0.08)	53.6 (0.74)	32.8 (0.82)	16.9 (0.88)	6.7 (0.91)	2.0 (0.77)
λ_{trace}^{BC}	278.1 (0.00)	207.8 (0.00)	140.7 (0.02)	86.3 (0.41)	45.9 (0.94)	28.1 (0.95)	13.5 (0.97)	5.7 (0.95)	1.27 (0.90)
	weak exogeneity								
	q	Δp	Δpe	i^l	ie^l	i^s	ie^s	cds	$cdse$
$r=4$	17.2 (0.00)	27.9 (0.00)	39.5 (0.00)	30.2 (0.00)	4.8 (0.31)	19.1 (0.00)	16.4 (0.00)	17.6 (0.00)	36.5 (0.00)
	cointegration rank, conditional model*								
$p-r$	8	7	6	5	4	3	2	1	
λ_i	0.46	0.42	0.37	0.30	0.13	0.10	0.04	0.02	
λ_{max}	168.9	136.7	85.7	63.1	23.9	16.4	5.1	2.8	
λ_{trace}	305.6 (0.00)	222.4 (0.00)	148.7 (0.00)	87.0 (0.06)	40.3 (0.81)	21.4 (0.90)	7.8 (0.97)	2.8 (0.86)	
λ_{trace}^{BC}	266.5 (0.00)	195.3 (0.00)	130.9 (0.05)	76.0 (0.28)	35.0 (0.94)	18.5 (0.97)	6.6 (0.99)	2.5 (0.89)	
	weak exogeneity, conditional model*								
	q	Δp	Δpe	i^l	i^s	ie^s	cds	$cdse$	
$r=4$	17.5 (0.00)	32.7 (0.00)	39.6 (0.00)	29.3 (0.00)	18.6 (0.00)	21.1 (0.00)	22.2 (0.00)	42.9 (0.00)	

Notes: λ_i denotes eigenvalues, the *p*-values (for simulated asymptotic distribution) are parenthesized.

* Estimates from the partial model (Harbo *et al.*, 1998; Ericsson *et al.*, 1998), ie^l assumed to be weakly exogenous.

TABLE 3

Inference on the long-run structure

	q	Δp	Δpe	i^l	ie^l	i^s	ie^s	cds	$cdse$	*p-value*
$\mathcal{H}_1$	0	-1	0	1	0	0	0	0	0	0.02
$\mathcal{H}_2$	0	-1	0	0	0	1	0	0	0	0.00
$\mathcal{H}_3$	0	0	-1	0	1	0	0	0	0	0.01
$\mathcal{H}_4$	0	0	-1	0	0	0	1	0	0	0.00
$\mathcal{H}_5$	0	-1	1.36	1	-1.36	0	0	0	0	0.70
$\mathcal{H}_6$	0	-1	4.80	0	0	1	-4.80	0	0	0.00
$\mathcal{H}_7$	0	0	0	1	0	-1	0	0	0	0.00
$\mathcal{H}_8$	0	0	0	0	1	0	-1	0	0	0.00
$\mathcal{H}_9$	0	0	0	1	18.76	-1	-18.76	0	0	0.00
$\mathcal{H}_{10}$	0	-1.23	0	-1	0	1	0	0	0	0.27
$\mathcal{H}_{11}$	0	-1	0	-1	0	1	0	0	0	0.22
$\mathcal{H}_{12}$	0	0	-21.83	0	-1	0	1	0	0	0.69
$\mathcal{H}_{13}$	0	0	-1	0	-1	0	1	0	0	0.28
$\mathcal{H}_{14}$	0.008	-1	1.58	1	-1.58	0	0	0	0	0.79
$\mathcal{H}_{15}$	0.005	-1	1	1	-1	0	0	0	0	0.74
$\mathcal{H}_{16}$	0.002	-1	0.91	1	-0.91	0	0	-0.001	0.001	0.69
$\mathcal{H}_{17}$	0.001	-1	1	1	-1	0	0	-0.001	0.001	0.80
$\mathcal{H}_{18}$	0.142	0	0	1	-1	0	0	0	0	0.00
$\mathcal{H}_{19}$	-0.033	0	0	1	-1	0	0	-0.005	0.005	0.34
$\mathcal{H}_{20}$	0	1	-0.37	-0.63	0	0	0	0	0	0.99

Notes: Intercepts in all cointegrating relations. Estimates from the partial model.

TABLE 4

The loadings matrix

	$\hat{\alpha}_1'$							
	Δq	$\Delta^2 p$	$\Delta^2 pe$	Δi^l	Δi^s	Δie^s	Δcds	$\Delta cdse$
$\hat{\alpha}_{1,1}'$	**-4.71** (-2.0)	0.27 (1.0)	0.05 (0.3)	**0.10** (3.9)	**-0.05** (-2.4)	-0.01 (-0.6)	16.23 (0.9)	-10.28 (-0.6)
$\hat{\alpha}_{1,2}'$	0.94 (0.5)	0.32 (1.6)	**0.82** (6.1)	-0.04 (-1.5)	0.02 (0.7)	**-0.02** (-2.0)	-19.09 (-1.5)	7.23 (0.5)
$\hat{\alpha}_{1,3}'$	**-0.09** (-4.0)	**-0.01** (-2.7)	0.00 (-1.6)	0.00 (0.5)	0.00 (-0.9)	0.00 (-0.3)	**-0.65** (4.0)	**0.82** (4.9)
$\hat{\alpha}_{1,4}'$	-4.41 (-1.7)	**-0.27** (-2.9)	-0.08 (-0.4)	**0.11** (5.0)	0.02 (0.4)	0.00 (-0.2)	19.52 (1.0)	-2.01 (-0.1)

Notes: The significant coefficients are bolded.

TABLE 5

The MA representation, β restricted

					$\hat{\alpha}'_{\perp}$				
	$\hat{\varepsilon}_{q}$	$\hat{\varepsilon}_{\Delta p}$	$\hat{\varepsilon}_{\Delta pe}$	$\hat{\varepsilon}_{i^l}$	$\hat{\varepsilon}_{ie^l}$	$\hat{\varepsilon}_{i^s}$	$\hat{\varepsilon}_{ie^s}$	$\hat{\varepsilon}_{cds}$	$\hat{\varepsilon}_{cdse}$
$\hat{\alpha}'_{\perp,1}$	-0.11	0.30	-0.94	0.04	0.04	-0.07	0.03	-0.04	0.02
$\hat{\alpha}'_{\perp,2}$	0.02	0.07	0.07	0.27	0.23	-0.03	0.93	0.00	0.00
$\hat{\alpha}'_{\perp,3}$	0.01	-0.08	0.05	0.22	-0.06	-0.97	-0.08	0.00	0.00
$\hat{\alpha}'_{\perp,4}$	-0.01	-0.06	0.01	-0.31	0.93	-0.11	-0.14	0.00	0.00
$\hat{\alpha}'_{\perp,5}$	0.05	0.21	0.08	0.86	0.27	0.19	-0.33	0.01	0.00
					normalized $\hat{\alpha}'_{\perp}$				
$\hat{\alpha}'_{\perp,1}$	0.00 (-)	-34.60 (-1.1)	35.39 (1.5)	-95.89 (-0.7)	0.00 (-)	0.00 (-)	0.00 (-)	1.00 (-)	**-0.95** (-3.0)
$\hat{\alpha}'_{\perp,2}$	0.00 (-)	0.00 (-0.1)	0.03 (1.5)	-0.07 (-0.7)	0.00 (-)	0.00 (-)	1.00 (-)	0.00 (-)	0.00 (0.6)
$\hat{\alpha}'_{\perp,3}$	0.00 (-)	0.17 (1.8)	-0.09 (-1.2)	0.08 (0.2)	0.00 (-)	1.00 (-)	0.00 (-)	0.00 (-)	0.00 (1.5)
$\hat{\alpha}'_{\perp,4}$	0.00 (-)	-0.04 (-1.3)	0.02 (1.0)	**-0.26** (-2.0)	1.00 (-)	0.00 (-)	0.00 (-)	0.00 (-)	0.00 (-0.5)
$\hat{\alpha}'_{\perp,5}$	1.00 (-)	9.85 (1.5)	-4.47 (-0.9)	34.46 (1.2)	0.00 (-)	0.00 (-)	0.00 (-)	0.00 (-)	**0.17** (2.5)
					$\hat{\tilde{\beta}}'_{\perp}$				
	q	Δp	Δpe	i^l	ie^l	i^s	ie^s	cds	$cdse$
$\hat{\tilde{\beta}}'_{\perp,1}$	**-0.12** (-3.4)	-0.00 (-1.5)	0.00 (-1.8)	0.00 (-1.2)	0.00 (-1.5)	0.00 (-1.3)	**-0.00** (-2.6)	**2.17** (4.3)	**1.26** (2.9)
$\hat{\tilde{\beta}}'_{\perp,2}$	26.53 (0.8)	0.98 (1.7)	**2.52** (6.1)	0.24 (0.3)	-0.27 (-0.9)	1.22 (0.9)	**2.25** (4.7)	72.34 (0.2)	184.5 (0.5)
$\hat{\tilde{\beta}}'_{\perp,3}$	-9.97 (-0.5)	**0.81** (2.6)	-0.16 (-0.7)	**1.28** (3.0)	0.31 (1.8)	**2.09** (2.8)	0.15 (0.6)	4.00 (0.0)	-264.6 (-1.2)
$\hat{\tilde{\beta}}'_{\perp,4}$	-15.16 (-0.6)	-0.29 (-0.7)	**-1.40** (-4.8)	0.24 (0.4)	**1.12** (5.1)	-0.05 (-0.1)	-0.28 (-0.8)	278.0 (0.9)	327.7 (1.2)
$\hat{\tilde{\beta}}'_{\perp,5}$	0.05 (0.1)	**-0.01** (-2.2)	-0.01 (-1.7)	**-0.01** (-2.0)	0.00 (-0.7)	**-0.03** (-2.1)	-0.01 (-1.9)	8.21 (1.9)	**10.93** (2.9)
					$\hat{C}$				
	$\hat{\varepsilon}_{q}$	$\hat{\varepsilon}_{\Delta p}$	$\hat{\varepsilon}_{\Delta pe}$	$\hat{\varepsilon}_{i^l}$	$\hat{\varepsilon}_{ie^l}$	$\hat{\varepsilon}_{i^s}$	$\hat{\varepsilon}_{ie^s}$	$\hat{\varepsilon}_{cds}$	$\hat{\varepsilon}_{cdse}$
q	0.05 (0.1)	**3.57** (2.0)	-3.41 (-1.5)	14.87 (1.1)	-15.16 (-0.6)	-9.97 (-0.5)	26.53 (0.8)	**-0.12** (-3.4)	**0.12** (5.5)
Δp	**-0.01** (-2.2)	**0.07** (2.2)	-0.04 (-1.0)	-0.24 (-1.0)	-0.29 (-0.7)	**0.81** (2.6)	0.98 (1.7)	-0.00 (-1.5)	0.00 (0.7)
Δpe	-0.01 (-1.7)	-0.01 (-0.6)	0.04 (1.6)	0.04 (0.2)	**-1.40** (-4.8)	-0.16 (-0.7)	**2.52** (6.1)	0.00 (-1.8)	0.00 (0.0)
i^l	**-0.01** (-2.0)	**0.10** (2.5)	-0.08 (-1.5)	-0.38 (-1.1)	0.24 (0.4)	**1.28** (3.0)	0.24 (0.3)	0.00 (-1.2)	0.00 (0.7)
ie^l	0.00 (-0.7)	0.01 (0.5)	-0.02 (-0.9)	**-0.27** (-2.1)	**1.12** (5.1)	0.31 (1.8)	-0.27 (-0.9)	0.00 (-1.5)	**0.00** (2.0)
i^s	**-0.03** (-2.1)	**0.17** (2.4)	-0.11 (-1.3)	-0.62 (-1.1)	-0.05 (-0.1)	**2.09** (2.8)	1.22 (0.9)	0.00 (-1.3)	0.00 (0.7)
ie^s	-0.01 (-1.9)	-0.01 (-0.2)	0.03 (0.8)	-0.24 (-1.7)	-0.28 (-0.8)	0.15 (0.6)	**2.25** (4.7)	**-0.00** (-2.6)	0.00 (1.3)
cds	8.21 (1.9)	-4.66 (-0.2)	48.24 (1.6)	-2.93 (0.0)	278.0 (0.9)	4.00 (0.0)	72.34 (0.2)	**2.17** (4.3)	**-0.71** (-2.4)
$cdse$	**10.93** (2.9)	5.44 (0.3)	32.12 (1.2)	136.8 (0.8)	327.7 (1.2)	-264.6 (-1.2)	184.5 (0.5)	**1.26** (2.9)	0.24 (0.9)
					Residual standard deviation				
	0.0213	0.0004	0.0003	0.0005	0.0002	0.0009	0.0003	0.2944	0.2552

Notes: The significant coefficients are bolded.

TABLE 6

The MA representation, α and β restricted

	normalized $\hat{\alpha}'_{\perp}$								
	$\hat{\varepsilon}_{q}$	$\hat{\varepsilon}_{\Delta p}$	$\hat{\varepsilon}_{\Delta pe}$	$\hat{\varepsilon}_{i^l}$	$\hat{\varepsilon}_{ie^l}$	$\hat{\varepsilon}_{i^s}$	$\hat{\varepsilon}_{ie^s}$	$\hat{\varepsilon}_{cds}$	$\hat{\varepsilon}_{cdse}$
$\hat{\alpha}'_{\perp,1}$	0.00	-43.91	41.00	-159.1	0.00	0.00	0.00	1.00	**-0.97**
	(-)	(-1.2)	(1.5)	(-1.0)	(-)	(-)	(-)	(-)	(-2.9)
$\hat{\alpha}'_{\perp,2}$	0.00	0.01	0.02	0.04	0.00	0.00	1.00	0.00	0.00
	(-)	(0.5)	(0.8)	(0.3)	(-)	(-)	(-)	(-)	(0.8)
$\hat{\alpha}'_{\perp,3}$	0.00	0.18	-0.10	0.15	0.00	1.00	0.00	0.00	0.00
	(-)	(1.7)	(-1.2)	(0.3)	(-)	(-)	(-)	(-)	(1.5)
$\hat{\alpha}'_{\perp,4}$	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)	(-)
$\hat{\alpha}'_{\perp,5}$	1.00	10.32	-4.77	37.92	0.00	0.00	0.00	0.00	**0.17**
	(-)	(1.4)	(-0.9)	(1.2)	(-)	(-)	(-)	(-)	(2.5)
	$\hat{\tilde{\beta}}'_{\perp}$								
	q	Δp	Δpe	i^l	ie^l	i^s	ie^s	cds	$cdse$
$\hat{\tilde{\beta}}'_{\perp,1}$	**-0.12**	0.00	0.00	0.00	0.00	0.00	**-0.00**	**2.15**	**1.33**
	(-3.0)	(-1.4)	(-1.8)	(-1.0)	(-0.6)	(-1.2)	(-2.1)	(4.0)	(2.7)
$\hat{\tilde{\beta}}'_{\perp,2}$	27.44	0.98	**2.54**	0.24	-0.29	1.23	**2.25**	63.52	171.0
	(0.8)	(1.9)	(6.0)	(0.3)	(-0.9)	(1.0)	(4.9)	(0.1)	(0.4)
$\hat{\tilde{\beta}}'_{\perp,3}$	-16.98	**0.70**	-0.09	**1.07**	-0.01	**1.77**	-0.09	39.76	-305.9
	(-0.8)	(2.1)	(-0.3)	(2.4)	(0.0)	(2.3)	(-0.3)	(0.1)	(-1.2)
$\hat{\tilde{\beta}}'_{\perp,4}$	-8.84	-0.18	**-1.48**	0.44	**1.43**	0.26	-0.04	243.6	371.7
	(-0.3)	(-0.4)	(-4.1)	(0.7)	(5.2)	(0.2)	(-0.1)	(0.6)	(1.1)
$\hat{\tilde{\beta}}'_{\perp,5}$	0.11	**-0.01**	-0.01	**-0.01**	0.00	**-0.02**	-0.01	7.77	**11.23**
	(0.3)	(-2.0)	(-1.7)	(-2.0)	(0.2)	(-2.0)	(-1.5)	(1.7)	(2.74)

Notes: The significant coefficients are bolded.